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Nonlinear Robust Controller Design for Plasma Magnetic Control in Fusion Reactors

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Abstract. Tokamak is a fusion reactor that provides extremely high temperature needed for fusion process. Such a temperature is created by confining plasma via some poloidal coils. Control of voltages to be applied to the poloidal coils is an important control problem for proper operation of the overall system. This paper deals with the nonlinear robust control of plasma current, shape, and position in a Tokamak reactor. The paper briefly discusses the modeling issue of a Tokamak, and then, based on a nonlinear state-space model, presents three different versions of a robust controller for the process. All parameters of the system are assumed to be unknown for all three types of the controllers. Lyapunov-type arguments are used to prove the stability of the proposed control schemes. Advantages and disadvantages with each type of controllers are discussed in detail.

1. Introduction

Tokamak is a very powerful reactor to produce a very high temperature, approximately 100×10^6 °C [1], needed for fusion process. Main function of this reactor is to confine the plasma by using some poloidal coils. Controlled voltages are applied to poloidal coils and this creates currents that produce the magnetic field. The mutual interaction between the currents and magnetic fields ensures that the plasma is positioned where heating is more efficient [2]. Well-known illustration of a Tokamak reactor, illustrated by European Fusion Development Agreement (EFDA) can be found in [3].

Feedback control is used to adjust the voltages of poloidal coils and, consequently, plasma current and plasma distance to wall. A designer has three options for designing a feedback controller for the system; (i) Linear controller, (ii) Nonlinear controller with exact model knowledge, and (iii) Nonlinear controller with parameter uncertainty.

Dynamic model of a Tokamak is highly nonlinear. A linear controller needs a linearized form of this model. But stability result of a linear controller for a nonlinear system is valid only at a small neighborhood of the equilibrium points of the states. Moreover, linearization may cancel some useful nonlinearities such as having quenching effects ones. On the other hand, linear control has a mature literature and many successful tools are available for designing a linear controller. For example, Belyakov and Kavin produces a linear model for control purposes in [4]. In [5], a very simple and control-oriented model is derived, and a linear robust controller is presented in [6]. Some other linear controllers for various types of Tokamak devices (JET, DIII-D, JT-60U) can be found in [7-14].



Nonlinear controllers, on the other hand, does not use any linearization method and, for this reason, do not have the drawbacks mentioned above. A failure in taking into account of any part of such a complex fusion reactor dynamics may lead a serious damage to the devices [15]. Designing and implementing a nonlinear controller is a good solution to avoid such a risk. Sharma et al is derived a nonlinear model for the Tokamak devices based on the classical arguments of Hamiltonian mechanics, but then they derive a low-order model from it [16]. In this study, a general nonlinear model is presented and a nonlinear controller is designed for plasma magnetic control in a fusion reactor. The rest of the paper is organized as follows: Section 2 briefly discusses the modeling issues and then defines the control problem. Section 3 presents nonlinear control design for the system and also presents stability analysis of the controller. The last section highlights some concluding remarks.

2. Problem Statement

Dynamic model of a Tokamak is characterized by dynamic behaviors of externally excited poloidal field coils, induced eddy currents, plasma, and passive structure [17]. Before presenting a state-space model, some important assumption listed below should be addressed [2]:

- The plasma circuit system in a Tokamak is assumed to be axisymmetric. This assumption is discussed in detail in [18].
- The plasma current profile can be described by means of a finite number of global parameters [19].
- At the time scale of interest for current, position, and shape control, inertial effects can be neglected due to the low plasma mass density. Therefore, the plasma can be assumed to be in equilibrium at each time instant [2].
- Skin currents flowing on the plasma boundary can be neglected [20].

Considering these assumptions, circuit equations for the Tokamak model can be written as follows;

$$\begin{aligned} \frac{d(L_e I_e + M_{es} I_s + REI_e)}{dt} + \Omega_e I_e &= V_e \\ \frac{d(L_s I_s + M_{se} I_e)}{dt} + \Omega_s I_s &= V_s \\ \frac{d(m_e \dot{R})}{dt} &= \frac{1}{2} I_e^T \frac{\partial L_e}{\partial R} I_e + I_s^T \frac{\partial M_{se}}{\partial R} I_e + \frac{1}{2} EI_e^2 \\ \frac{d(m_e \dot{z})}{dt} &= \frac{1}{2} I_e^T \frac{\partial L_e}{\partial z} I_e + I_s^T \frac{\partial M_{se}}{\partial z} I_e \end{aligned} \quad (1)$$

where I_e is plasma elements current, I_s represents structure current, R, z depict plasma position at R - and z -axes of cylindrical coordinate system, L_e is plasma elements inductance, L_s stands for structure inductance, M_{se}, M_{es} are mutual inductances, Ω_e is plasma elements resistance, Ω_s represents structure resistance, E depicts internal energy coefficient, V_e stands for plasma elements voltage and V_s is poloidal coil voltage. System model given in (1) can be written in a more compact form as

$$\dot{x} = f(x, \theta) - g(x, \theta)u(t) \quad (2)$$

where the state vector is

$$x = [I_e \quad I_s \quad \dot{r}]^T, \quad r = [R \quad z]^T \quad (3)$$

and the control input is

$$u = [V_e \quad V_s \quad 0] \quad (4)$$

and, finally, θ is the parameter vector containing constant parameters in (1). Note that

$$g(x, \theta) = I. \quad (5)$$

The generalized model given in (2) assumes that the system is linearly parameterizable. Such a parameterized form can be found in [16]. Control objective is to drive the components of the state vector, x , to a desired trajectory, x_d . To investigate the performance of the controller to be designed, a tracking error signal can be defined as

$$e = x_d - x. \quad (6)$$

It is assumed that the desired trajectory, x_d , is bounded with at least its second derivative.

3. Control Design

This section explains the design of a robust controller with three different versions to the control problem defined above. The main idea is to model the parameter uncertainties as a disturbance to the system and design the control signal so that the output is robust to these disturbances [21]. Investigating the corresponding open-loop error system dynamics yields

$$\dot{e} = \dot{x}_d - f(x, \theta) + u(t) \quad (7)$$

and, then, the robust controller formulation will be

$$u(t) = -\dot{x}_d - Ke + \hat{f} - V_R \quad (8)$$

where \hat{f} is a function formed by the estimates of uncertain parameters vector, θ , and K is a positive definite control gain matrix. An estimation error signal can be defined as

$$\tilde{f} \triangleq f - \hat{f} \quad (9)$$

It is assumed that the estimation error function can be bounded by a known function, $\rho(\cdot)$, as

$$\|\tilde{f}(x, \theta)\| \leq \rho(x, \hat{\theta}) \quad (10)$$

where $\hat{\theta}$ represents a vector containing the estimates of the elements of uncertain parameters vector, θ . In (8), V_R depicts the robust part of the controller to be designed. In this study, two versions for the robust part, that are, high frequency type and high gain type, are designed and presented in the followings. Then a third type of controller is introduced. Before presenting the design, expression for the error system dynamics at this step can be written as

$$\dot{e} = -Ke - \tilde{f} - V_R \quad (11)$$

3.1. High Frequency Type

Designing the robust part of the controller, V_R , as

$$V_R = \frac{\rho^2 e}{\|\rho\| \|e\| + \varepsilon} \quad (12)$$

where $\varepsilon > 0$ is a small constant, yields a closed-loop error dynamics in the form of

$$\dot{e} = -Ke - \tilde{f} - \frac{\rho^2 e}{\|\rho\| \|e\| + \varepsilon}. \quad (13)$$

Following theorem and its proof can be used to analyze the effect of the composite control input signal given in (8) and (12) on the stability of the system given in (1).

Theorem 1: For the system given in (1), if the control input is designed as in (8) and (12), then the tracking error converges to a bound like

$$|e(t)| \leq \sqrt{A + B \exp\{-2\lambda_{\min}\{K\}t\}} \quad (14)$$

with an exponential envelope, where $\exp\{\cdot\}$ is the natural logarithm function, $\lambda_{\min}\{K\}$ is the smallest eigenvalue of K , and

$$A = \frac{\varepsilon}{\lambda_{\min}\{K\}}, \quad B = |e(0)|^2 - \frac{\varepsilon}{\lambda_{\min}\{K\}}. \quad (15)$$

Proof 1: To prove the theorem, a positive definite, scalar function can be used;

$$V = \frac{1}{2} e^T e. \quad (16)$$

Taking the time derivative of this function yields

$$\dot{V} = e^T \left\{ -Ke - \tilde{f} - \frac{\rho^2 e}{\|\rho\| \|e\| + \varepsilon} \right\} \quad (17)$$

which can be upper bounded as

$$\begin{aligned} \dot{V} &\leq -\lambda_{\min}\{K\} \|e\|^2 + \|\rho\| \|e\| - \frac{\rho^2 \|e\|}{\|\rho\| \|e\| + \varepsilon} \\ &\leq -\lambda_{\min}\{K\} \|e\|^2 + \varepsilon \left[\frac{\rho \|e\|}{\rho \|e\| + \varepsilon} \right] \leq -\lambda_{\min}\{K\} \|e\|^2 + \varepsilon. \end{aligned} \quad (18)$$

Considering (16) and (18) together, one can write

$$V(t) \leq V(0) \exp\{-\gamma t\} + \frac{\varepsilon}{\gamma} [1 - \exp\{-\gamma t\}] \quad (19)$$

where $V(0)$ is initial value of V , and

$$\gamma \triangleq 2\lambda_{\min}\{K\} \quad (20)$$

Then $e(t)$ can be upper bounded as in (14) and this completes the proof.

3.2. High Gain Type

For this type, the robust part of the controller is designed as

$$V_R = \frac{e\rho^2}{\varepsilon} \quad (21)$$

so that the corresponding closed-loop error system dynamics becomes

$$\dot{e} = -Ke - \frac{e\rho^2}{\varepsilon} - \tilde{f}. \quad (22)$$

Then the following theorem and its proof can be used to analyze the effect of the composite control input signal given in (8) and (21) on the stability of the system given in (1).

Theorem 2: The stability result given in Theorem 1 is also valid for the system if the control input is designed as in (8) and (21).

Proof 2: By using the same Lyapunov function given in (16) and taking again its time derivative, one gets

$$\dot{V} = -e^T Ke - \frac{\rho^2 e^T e}{\varepsilon} + e^T \tilde{f} \quad (23)$$

which can be upper bounded as

$$\dot{V} \leq -\lambda_{\min}\{K\} \|e\|^2 + \rho \|e\| - \frac{\rho^2 \|e\|}{\varepsilon} \leq -\lambda_{\min}\{K\} \|e\|^2 + \rho \|e\| \left[1 - \frac{\rho \|e\|}{\varepsilon} \right]. \quad (24)$$

There exist two possible cases for the inequality given in (24).

Case 1: If $|\rho| \|e\| > \varepsilon$, then one can directly write

$$\dot{V} \leq -\lambda_{\min}\{K\} \|e\|^2. \quad (25)$$

Considering (16) and (25) together, it is inferred that $V(t)$ and its component, $e(t)$, goes to zero exponentially.

Case 2: If $\|\rho\| \|e\| < \varepsilon$, then

$$0 < \left(1 - \frac{\rho \|e\|}{\varepsilon}\right) \leq 1 \quad (26)$$

and

$$\dot{V} \leq -\lambda_{\min}\{K\} \|e\|^2 + \varepsilon \quad (27)$$

which gives same stability result in the high frequency type.

3.3. Smooth Robust Controller

Electrical drive systems used in the fusion reactors may suffer from chattering in system currents depending on the size of the reactor. Instant variations in the currents induce undesirable effects on fusion performance, since a linear amplifier cannot be used as the drive system due to the existence of high current requirement. Cooling such a linear amplifier would require some huge coolers with unreasonable dimensions. Instead, a more complex power electronics equipment are needed for drive system and these equipment generally suffer from rapid changes in the system currents created by chattering phenomena. Since this possibility is valid for all the systems with high current requirement, robust control researchers continuously seek more smooth robust schemes. For example, a useful topology is introduced in [22]. Tokamaks control systems may also require such topologies, again depending on the size of the reactor. This does not necessarily mean that high gain and high frequency type controllers introduced above are not useful and functional for Tokamaks. These controllers may still be successful for controlling plasma currents, shape, and position. Smooth robust topologies provide an alternative design approach to cope with some possible problems.

To investigate the smooth robust controller formulation for the system given in (1), let reconsider the error system dynamics given in (7), and define a new error variable, called filtered tracking error, as

$$\eta = \dot{e} + \alpha e \quad (28)$$

where $\alpha \in \mathbb{R}^+$ is a constant. Note that

$$e(s) = \frac{1}{s + \alpha} \eta(s) \quad (29)$$

which means that if $\eta(t)$ goes to zero, then $e(t)$ also goes to zero, and so does $\dot{e}(t)$, where s is Laplace variable. Investigating $\eta(t)$ dynamics yields

$$\begin{aligned} \dot{\eta} &= \ddot{e} + \alpha \dot{e} \\ &= \ddot{x}_d - \ddot{x} + \alpha \dot{e} = \ddot{x}_d + \alpha \dot{e} - \dot{f}(x, \theta) + \dot{u} - \dot{e} + \dot{e}. \end{aligned} \quad (30)$$

Let reconfigure $\eta(t)$ dynamics as

$$\dot{\eta} = N(\dot{x}, \theta) - e + \dot{u} \quad (31)$$

where

$$N(\dot{x}, \theta) = \ddot{x}_d + \alpha \dot{e} - \dot{f}(x, \theta) + e. \quad (32)$$

Now define

$$N_d \triangleq \ddot{x}_d + \frac{\partial f(x_d, \theta)}{\partial x_d} \dot{x}_d \quad (33)$$

in order to write

$$\dot{\eta} = \tilde{N} - e + \dot{u} + N_d, \quad \tilde{N} = N - N_d. \quad (34)$$

In (33), N_d represents the desired profile for N function. When the desired profile and its time derivatives are bounded, then N_d and its derivative are also bounded. Robust controller formulation assumes that \tilde{N} and \tilde{f} can be upper bounded by some functions, $\sigma(\cdot)$ and $\mu(\cdot)$, as

$$\begin{aligned} \|\tilde{N}\| &\leq \sigma(\|\varphi\|)\|\varphi\|, \quad \varphi \triangleq [e \quad \eta]^T \\ \|\tilde{f}(x, \theta)\| &\leq \mu(x, \hat{\theta}) \end{aligned} \quad (35)$$

where $\tilde{f} = f - \hat{f}$, \hat{f} is the estimation of f function, and $\hat{\theta}$ is the estimation of unknown parameters vector. Now one can design \dot{u} as

$$\dot{u} = -[(K_s + I)\eta + \beta \text{sgn}(e)] \quad (36)$$

where β and K_s are some positive control gain matrices and I is identity matrix. Then the closed-loop error system dynamics becomes

$$\dot{\eta} = -(K_s + I)\eta - \beta \text{sgn}(e) + N_d - e + \tilde{N}. \quad (37)$$

The implementable form of the actual control input signal, u , will be given at the end of the following stability analysis. The following theorem and its proof can be used to show the stability of the overall system.

Theorem 3: For the system given in (1), the control input signal defined by (37) ensures that the tracking error signal converges to as time goes to infinity, if the minimum eigenvalues of the control gain matrices are selected to satisfy

$$\lambda_{\min}\{\beta\} > \|N_d(t)\| + \frac{1}{\alpha} \|\dot{N}_d\|, \quad \text{and} \quad \lambda_{\min}\{K_s\} \geq \frac{1}{4} \sigma^2(\|\varphi\|). \quad (38)$$

The designed control input is also guarantees that all signals in the closed-loop system are bounded.

Proof 3: The proof is omitted due to lack of space.

4. Conclusion

Three different versions of a nonlinear robust controller for plasma current, shape, and position control in a nuclear fusion reactor were presented. The first type, high frequency controller, drives the plasma current, shape, and position to a desired trajectory with an adjustable error, while the second type, high gain controller, may drive the tracking error signal to zero by condition, or, at least to an adjustable bound. If the worst case of these controllers, that is, the tracking error signal converges to an adjustable bound, is occurred, this may lead chattering in poloidal coils voltage and plasma elements voltage, and in currents created by these voltages. But, still the control objective is achieved. The third controller provides an alternative robust control strategy for the reactor and aims to reduce chattering. All three controllers provide global stability result, that is, global confinement of plasma is achieved.

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