Journal of Algebra and Its Applications
(2019) 1950062 (10 pages)

DOI: 10.1142/S0219498819500622

Full idempotents in Leavitt path algebras

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Received 10 August 2017
Accepted 13 March 2018
Published 7 June 2018

Communicated by P. Ara

We give necessary and sufficient conditions on a directed graph E for which the associated Leavit path algebra $L_K(E)$ has at least one full idempotent. Also, we define $E_n, n \geq 0$ sub-graphs of E and show that $L_K(E)$ has at least one full idempotent if and only if there is a sub-graph E_r such that the associated Leavitt path algebra $L_K(E_r)$ has at least one full idempotent.

Keywords: Full idempotent; Leavitt path algebra; restriction graph; Morita invariant property; source elimination.

Mathematics Subject Classification: 16S99, 16D90, 05C25

1. Introduction

In [10, Theorem 3], it was shown that, for any directed graph E and for any commutative ring R with identity, there is a subset $V \subset E^0$ by which we can define a Leavitt path algebra such that it is Morita equivalent to $L_R(E)$. In this paper, for any directed graph E and any field K, we give necessary and sufficient conditions for which there is subset $V \subset E^0$ by which we can define a full idempotent in $L_K(E)$.

Before explaining the idea behind this work, we give some definitions. Let R be any ring. For any idempotent $e \in R$, the ring eRe is said to be a corner ring of R. Clearly eRe is a ring with identity e. If $ReR = R$, then e is said to be a full idempotent in R. Let E be a directed graph and K any field, then Leavitt path algebra associated to E and K is denoted by $L_K(E)$. In the Preliminaries section, we give necessary information on Leavitt path algebras (for more information and relevant terminology on Leavitt path algebras, see [3, 4]). Now here is the idea behind this work: Suppose that P and Q are two properties defined on rings such that, for any ring R with identity, if R satisfies P, then it does Q. Now assume that P and Q are Morita invariant properties (for more information on Morita invariant properties, see [13, 14]). In this case even if R does not have an identity but has a full idempotent e and satisfies P, then it satisfies Q. To see that recall, by [1, Corollary 4.3], that R and eRe are Morita equivalent rings. Since P is Morita invariant and eRe is a ring with identity e, eRe satisfies P and so does Q. Therefore since Q is Morita invariant, R satisfies Q. Hence, if the properties P and Q are Morita invariant, then instead of having a unit to satisfy Q, it is sufficient for R to satisfy Q that R has a full idempotent. So we can use that for the generalization such as in [7, Corollary 1.2].

This paper is divided into three sections. In the Preliminaries section, we give some graph-theoretic definitions and properties. Then in the main section of this paper, we define equivalence vertices, maximal vertices, maximal set of a hereditary subset $H \subseteq E^0$ and the set $\mathcal{H}^E$ which consists of hereditary subsets having some properties and, as the first main result of this paper, we prove that, for any directed graph E and any field K, $L_K(E)$ Leavitt path algebra has at least one full idempotent if and only if there is at least one $H \in \mathcal{H}^E$ such that the maximal set of H is finite (Theorem 3.1). Then by using the restriction graphs defined in [8, p. 3], we define $E_n, n \geq 0$ subgraphs of E (Definition 3.5) and prove the second main result of this paper that $L_K(E)$ has at least one full idempotent if and only if there is a sub-algebra $L_K(E_r), r \geq 0$ of $L_K(E)$ such that $L_K(E_r)$ has at least one full idempotent (Theorem 3.2). Thus, we simplify the problem whether or not $L_K(E)$ has a full idempotent. Furthermore, in Lemma 3.4, by using a different approach we prove that, for any directed graph E, if we denote by F the directed graph obtained by applying source elimination to E, then Leavitt path algebra over E is Morita equivalent to the Leavitt path algebra associated to the one over F. Finally, in the last section of this paper, we give some examples.

2. Preliminaries

We briefly recall some graph-theoretic definitions and properties. For more information see [2]. A (directed) graph $E = (E^0, E^1, r, s)$ consists of two arbitrary sets E^0, E^1 and maps $r, s : E^1 \to E^0$. The elements of E^0 are called vertices and the elements of E^1 edges. If $s^{-1}(v)$ is a finite set for every $v \in E^0$, then the graph is called *row-finite*. A vertex for which $s^{-1}(v)$ is empty is called a sink, a vertex for which $r^{-1}(v)$ is empty is called a source and, a vertex $v \in E^0$ for which $|s^{-1}(v)| = \infty$ is called an infinite emitter. If v is either a sink or an infinite emitter, we call it a singular vertex. If v is not a singular vertex, we call it a regular vertex. A path μ in a graph E is a sequence of edges $\mu = e_1, \dots, e_n$ such that $r(e_i) = s(e_{i+1})$ for $i = 1, \dots, n-1$. In this case, $s(\mu) := s(e_1)$ is the source of μ, $r(\mu) := r(e_n)$ is the range of μ, and n is the length of μ. If $\mu = e_1 \dots e_n$ is a path, then we denote by μ^0 the set of its vertices, that is, $\mu^0 = \{s(e_1), r(e_i) \text{ for } 1 \leq i \leq n\}$. If μ is a path in E, and if $v = s(\mu) = r(\mu)$, then μ is called a closed path based at v. If $s(\mu) = r(\mu)$ and $s(e_i) \neq s(e_j)$ for every $i \neq j$, then μ is called a cycle. For $n \geq 2$, we define

E^n to be the set of paths of length n, and $E^* = \bigcup_{n \geq 0} E^n$ the set of all paths. The Leavitt path algebra of a graph E is defined as the following.

Let E be any directed graph, and K any field. The Leavitt path K-*algebra* $L_K(E)$ of E with coefficients in K is the K-*algebra* generated by a set $\{v | v \in E^0\}$ of pairwise orthogonal idempotents, together with a set of variables $\{e, e^* | e \in E^1\}$, which satisfy the following relations:

(1) $s(e)e = er(e) = e$ for all $e \in E^1$.
(2) $r(e)e^* = e^* s(e) = e^*$ for all $e \in E^1$.
(3) $e^* e' = \delta_{e,e'} r(e)$ for all $e, e' \in E^1$.
(4) $v = \sum_{\{e \in E^1 | s(e) = v\}} ee^*$ for every regular vertex $v \in E^0$.

The elements of E^1 are called real edges, while for $e \in E^1$ we call e^* a ghost edge. The set $\{e^* | e \in E^1\}$ will be denoted by $(E^1)^*$. We let $r(e^*)$ denote $s(e)$, and we let $s(e^*)$ denote $r(e)$. If $\mu = e_1 \dots e_n$ is a path, then we denote by μ^* the element $e_n^* \dots e_1^*$ of $L_K(E)$.

Specifically, we define a relation $\geq$ on E^0 by setting $v \geq w$ if there is a path $\mu \in E^*$ with $s(\mu) = v$ and $r(\mu) = w$. A subset H of E^0 is called hereditary if $v \geq w$ and $v \in H$ imply $w \in H$. Denote by $\mathcal{H}_E$ the set of hereditary subsets of E^0. A hereditary set is saturated if every regular vertex which feeds into H and only into H is again in H, that is, if v is a regular vertex such that $s^{-1}(v) \neq \emptyset$ and $r(s^{-1}(v)) \subseteq H$, then necessarily $v \in H$.

The hereditary saturated closure of a set X of vertices is defined as the smallest hereditary and saturated subset of E^0 containing X. In [9, Remark 3.1], it was shown that the hereditary saturated closure of a set X of vertices is $\overline{X} = \bigcup_{n=0}^{\infty} \Lambda_n(X)$, where $\Lambda_0(X) = T(X) = \{v \in E^0 : x \geq v \text{ for some } x \in X\}$, and $\Lambda_n(X) = \{y \in E^0 : 0 < |s^{-1}(y)| < \infty \text{ and } r(s^{-1}(y)) \subseteq \Lambda_{n-1}(X)\} \cup \Lambda_{n-1}(X)$, for $n \geq 1$, where the set $T(X)$ is called tree of X and clearly the smallest hereditary subset of E^0 containing X.

If E^0 is finite, then we have $\sum_{v \in E^0} v = 1$; otherwise, $L_K(E)$ is a ring with a set of local units consisting of sums of distinct vertices. Conversely, if $L_K(E)$ is unital, then E^0 is finite.

3. Full Idempotents in Leavitt Path Algebras

Before proving the first main result of this paper, we need some definitions and remarks.

Definition 3.1. Let E be a directed graph. If for two vertices u and v, it is true that $u \geq v$ and $v \geq u$, then we call u and v as equivalence vertices and denote by $u \approx v$.

It is straightforward to show that "$\approx$" is an equivalence relation on E^0.

Definition 3.2. Let E be a directed graph and let $X \subseteq E^0$. If there is a vertex u in X such that whenever $v \geq u$, so $u \geq v$ for all $v \in X$, then u is called maximal in X.

Moreover, if $H \subseteq E^0$ is hereditary subset, then the maximal set of H is defined as:

$$\overrightarrow{H} = \{[u] : u \,\text{is maximal in}\, H\}.$$

Example 3.1.

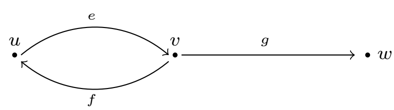

Fig. 1. E.

For the graph above, we have $u \approx v$ and $\overrightarrow{E^0} = \{[u]\}$.

Definition 3.3. Let E be a directed graph. We define the set $\mathcal{H}^E$ as:

$$\mathcal{H}^E = \left\{ H \in \mathcal{H}_E : H = \bigcup_{[u] \in \overrightarrow{H}} T(u) \;\text{ and }\; \overline{H} = E^0 \right\}.$$

It is clear that if $u \approx v$, then $T(u) = T(v)$. So the definition above is well defined.

Remark 3.1. Let E and F be directed graphs and K any field. Then for any subset X of E^0, we will denote by $I(X)$ the ideal of $L_K(E)$ generated by X. If $L_K(E)$ is Morita equivalent to $L_K(F)$, then we will briefly denote that by $L_K(E) \approx_M L_K(F)$.

Remark 3.2. Let E be a directed graph and suppose that $H \in E^0$ is a hereditary subset. Then since $T(H)$ is the smallest hereditary set containing H and H is hereditary subset of E^0, we get that $T(H) \subseteq H$, $H \subseteq T(H)$ and so $T(H) = H$.

Remark 3.3. Let E be e directed graph. If H is a subset of E^0, then $\overline{H} = I(H) \cap E^0$. (See the proof of [8, Lemma 2.1])

Now we are ready to prove the first main result of this article.

Theorem 3.1. *Let E be a directed graph and K a field. Then $L_K(E)$ has at least one full idempotent if and only if there is a set $H \in \mathcal{H}^E$ whose maximal set is finite.*

Proof. If $L_K(E)$ has a full idempotent e, then by [3, Lemma 1.6], there is an idempotent $u \in L_K(E)$ such that $u = \sum_{w \in W} w$ and $ue = eu = e$, where W is a finite subset of E^0. If we define the set $H = T(W) = \bigcup_{w \in W} T(w)$, then since W is finite, for every vertex $w \in W$, there is a maximal vertex v in W such that $w \in T(v)$. Therefore $H = \bigcup_{[v] \in \overrightarrow{H}} T(v)$ and clearly $\overrightarrow{H}$ is finite. Now we show that $\overline{H} = E^0$. Since $u \in I(H)$ and e is a full idempotent in $L_K(E)$, we have $e \in I(H)$ and $L_K(E) = I(e) \subseteq I(H)$. Therefore $I(H) = L_K(E)$ and so $\overline{H} = I(H) \cap E^0 = E^0$.

Conversely, suppose there is a set $H \in \mathcal{H}^E$ with a finite maximal set $V = \{v_1, v_2, \ldots, v_n\}$. If $I(V)$ is the ideal generated by V, then by the assumption on

H, $I(V) = I(H) = L_K(E)$. If $u = v_1 + v_2 + \cdots + v_n$, then u is an idempotent and $uv_i = v_i = v_iu$ and so $I(u) = I(V) = L_K(E)$. Thus, u is a full idempotent in $L_K(E)$. □

Corollary 3.1. *If e is a full idempotent in $L_K(E)$, then there is a full idempotent u which is a sum of vertices such that $I(u) = I(e)$.*

As a result of [12, Lemma 12], we can give the following lemma.

Lemma 3.1. *Let E be a directed graph and $H \subseteq E^0$ hereditary subset. If $u \in \overline{H}$ and u is a base for a closed path, then $u \in H$.*

Also by Theorem 3.1 and by Lemma 3.1, we can give the following two corollaries.

Corollary 3.2. *Let E be a directed graph and K a field. If $E^0 \notin \mathcal{H}^E$ and, for every H hereditary proper subset of E^0, there is a vertex u which is base of a closed path such that $u \notin H$, then $L_K(E)$ has no full idempotent.*

Proof. Assume that $E^0 \notin \mathcal{H}^E$ and, for every $H \subset E^0$ hereditary proper subset there is at least one vertex u which is base of a closed path such that $u \notin H$. Now suppose that $\mathcal{H}^E \neq \emptyset$. Then there is at least one $H \subseteq E^0$ hereditary subset such that $\overline{H} = E^0$. Then by the assumption, we have $H \neq E^0$. Therefore H must be a proper subset of E^0. But since for every proper hereditary subset H of E^0, there is a vertex u which is the base of a closed path such that $u \notin H$, by Lemma 3.1, we get that $\overline{H} \neq E^0$ and so $H \notin \mathcal{H}^E$, a contradiction. Therefore, we must have $\mathcal{H}^E = \emptyset$ and then by Theorem 3.1, $L_K(E)$ has no full idempotent. □

Corollary 3.3. *Let E be a directed graph and K a field. If there are infinitely many nonequivalent maximal vertices which are base of a closed path, then $L_K(E)$ has no full idempotent.*

Proof. Suppose that there are infinite number of nonequivalent maximal vertices in E which are the base of a closed path and denote by M the set of those vertices. If we assume that $L_K(E)$ has at least one full idempotent, then we can find a finite set V with similar way in the proof of Theorem 3.1. Since $\overline{V} = E^0$ and by Lemma 3.1, for every $u \in M$ it is true that $u \in T(V)$ and so there must be at least one $v \in V$ such that $v \geq u$. Hence because u is maximal, we get that $u \geq v$ and so $u \approx v$. Therefore if $u \in M$, then there is a vertex $v \in V$ such that u is equivalent to v. But this is impossible because V is a finite set and M consists of infinite number of nonequivalent vertices. Therefore $L_K(E)$ must not have any full idempotent. □

Now we recall restriction graph defined in [8, p. 3]. Let E be a directed graph and let H be hereditary subset of E^0. Then the restriction graph of E over H is defined as:

$$E_H = (H, \{e \in E^1 | s(e) \in H\}, r_{(E_H)^1}, s_{(E_H)^1}).$$

Proposition 3.1. *Let E be a directed graph and $H \subseteq J \subseteq K$ hereditary subsets of E^0. If hereditary saturated closure of H according to E_J is J and that of J according to E_K is K, then that of H according to E_K is K.*

Proof. We shall be using [6, Lemma 1.4] and the idea in its proof, namely, if $X \subseteq E^0$ is a nonempty hereditary set, then $Y \subseteq E^0$ is contained in the saturated closure $\overline{X}$ of X if and only if, for any regular vertex $u \in Y$, there is an integer $n \geq 0$ such that every path p in E with $s(p) = u$ and length $\geq n$ satisfies $r(p) \in X$. In particular, $r(q) \in X$ if q is any path of length exactly n in E with $s(q) = u$. We will also be using the fact that if u is any singular vertex in Y, then already $u \in X$. Consequently, all the singular vertices in $K = \overline{J}$ belong to J and are singular in J (as J is hereditary) and since $J = \overline{H}$, they all belong to H. In order to show that $\overline{H} = K$, let u be a regular vertex in K. Since $\overline{J} = K$ in E_K, there is an integer $m \geq 0$ such that every path p in E_K with $s(p) = u$ and length m satisfies that $r(p) \in J$. If $r(p)$ is singular, then already $r(p) \in H$. So consider only the paths p of length m, where $s(p) = u$ and $r(p)$ is regular and belongs to $J \backslash H$. In particular, since H is hereditary, every vertex in all these paths of length m must be regular. A simple induction on m then shows that there are only finitely many paths p of length m with $s(p) = u$ and $r(p) \in J \backslash H$. Let $p_1, \ldots, p_k$ be a listing of these paths of length m with $r(p_i) = v_i \in J, i = 1, \ldots, k$. Since $J = \overline{H}$, for each v_i, there is an integer $n_i \geq 0$ such that all paths q with $s(q) = v_i$ and length $\geq n_i$ satisfy $r(q) \in H$. Let $n = \max\{n_i : i = 1, \ldots, k\}$. Then every path μ in E_K with $s(\mu) = u$ and length $\geq m + n$ satisfies $r(\mu) \in H$. By Lemma 1.4 of [6], $K = \overline{H}$. □

Also by [6, Lemma 1.4], we can give the following lemma.

Lemma 3.2. *Let E be a directed graph and let H be a proper hereditary subset of E^0. Then $\overline{H} = E^0$ if and only if, for every $u \in E^0$, there is a positive integer n_u such that all paths which emit from u and whose length are at least n_u connect to H.*

Now we modify the source elimination process defined in [5, Definition 1.2].

Definition 3.4. Let $E = (E^0, E^1, r, s)$ be a directed graph. Denote by S the set of all regular sources in E and, define the following set:

$$s^{-1}(S) = \{e \in E^1 : e \in s^{-1}(v),\ v \in S\}.$$

Then we form the source elimination graph $E_{\backslash S}$ of E as follows:

$$E^0_{\backslash S} = E^0 \backslash S,$$

$$E^1_{\backslash S} = E^1 \backslash s^{-1}(S),$$

$$s_{E_{\backslash S}} = s|_{E^1_{\backslash S}},$$

$$r_{E_{\backslash S}} = r|_{E^1_{\backslash S}}.$$

Here we define $E_n, n \geq 0$ subgraphs, which we use to describe second main result of this paper, of E.

Definition 3.5. Let E be a directed graph. For $n \geq 0$, we define the sub-graphs E_n of E as: $E_0 = E$ and if $\mathcal{H}^{E_{n-1}} \neq \emptyset, n \geq 1$, then take any $H_{n-1} \in \mathcal{H}^{E_{n-1}}$, denote by G_{n-1} the restriction graph of E_{n-1} over H_{n-1} and take E_n as the result graph of applying source elimination to G_{n-1}.

In [5, Proposition 3.1], it was shown that removing sources from a row-finite graph would give a Leavitt path algebra Morita equivalent to the one associated to the original graph, and in [11], this result was generalized to any directed graph E. Now in Lemma 3.3, we prove that by using a different approach.

Lemma 3.3. *Let E be a directed graph. If F is the result graph of applying source elimination to E, then $\overline{F^0} = E^0$ and $L_K(E) \approx_M L_K(F)$.*

Proof. If we denote by $S \subseteq E^0$ the set of regular sources of E, then by Definition 3.4, we have $S \cup F^0 = E^0$ and $S = \{y \in E^0 : 0 < |s^{-1}(y)| < \infty,\ r(s^{-1}(y)) \subseteq F^0,\ y \notin F^0\}$. Therefore since F^0 is a hereditary subset of E^0, $\Lambda_1(F^0) = \{y \in E^0 : 0 < |s^{-1}(y)| < \infty,\ r(s^{-1}(y)) \subseteq \Lambda_0(F^0)\} \cup \Lambda_0(F^0) = \{y \in E^0 : 0 < |s^{-1}(y)| < \infty,\ r(s^{-1}(y)) \subseteq F^0\} \cup F^0 \supseteq S \cup F^0 = E^0$ and then $\overline{F^0} = E^0$. Since $E^0 = \overline{F^0} = I(F^0) \cap E^0$, we have $E^0 \subseteq I(F^0)$ and then $I(F^0) = L_K(E)$. Hence by [8, Lemma 2.4], we get that $L_K(F) = L_K(E_{F^0}) \approx_M I(F^0) = L_K(E)$. □

Lemma 3.4. *If we define a property P over any ring R as: "R has at least one full idempotent", then P is Morita invariant.*

Proof. Let R be a ring such that it satisfies P. If $e \in R$ is a full idempotent, then since the corner ring eRe has an identity, eRe has at least one full idempotent. Therefore by [13, Corollary 18.35], P is Morita invariant. □

Now we are ready to prove the second main result of this paper, so we restrict the problem of finding full idempotent in $L_K(E)$ to subalgebra $L_K(E_r), r \geq 0$ of $L_K(E)$.

Theorem 3.2. *Let E be a directed graph and K a field. Then $L_K(E)$ has at least one full idempotent if and only if there is a directed subgraph $E_r, r \geq 0$ of E such that $L_K(E_r)$ has at least one full idempotent.*

Proof. If $L_K(E)$ has at least one full idempotent, then $r = 0$. Now suppose that there is a subgraph $E_r, r \geq 0$ such that $L_K(E_r)$ has a full idempotent. If $r = 0$, then by $E_0 = E$, we have $L_K(E) \approx_M L_K(E_0)$. Suppose that $r \geq 1$. Then for $(E_r)^0 \subseteq (G_{r-1})^0 \subseteq (E_{r-1})^0$, we have $G_{r-1} = E_{H_{r-1}}$ and then $(G_{r-1})^0 = H_{r-1}$. So we get that $\overline{(G_{r-1})^0} = \overline{(H_{r-1})} = (E_{r-1})^0$ and, by Lemma 3.4, the saturated closure of $(E_r)^0$ according to G_{r-1} is $(G_{r-1})^0$. Therefore by Proposition 3.1, the saturated closure of $(E_r)^0$ according to E_{r-1} is $(E_{r-1})^0$. By continuing to repeat this process, we get that the saturated closure of $(E_r)^0$ according to $E_0 = E$ is E^0. Therefore since $E^0 = \overline{(E_r)^0} = I((E_r)^0) \cap E^0$, we get that $E^0 \subseteq I((E_r)^0)$ and then $I((E_r)^0) = L_K(E)$. Hence by [8, Lemma 2.4.], $L_K(E_r) = L_K(E_{(E_r)^0}) \approx_M$

$I((E_r)^0) = L_K(E)$ and so by Lemma 3.4, we get that $L_K(E)$ has a full idempotent, as desired. □

4. Examples

Example 4.1.

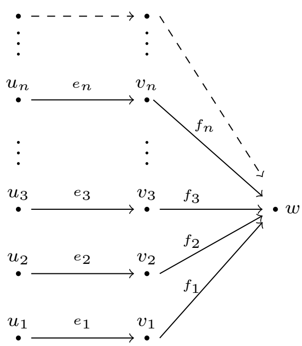

Fig. 2. E.

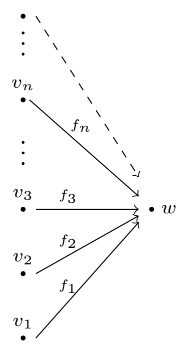

Fig. 3. E_1.

Fig. 4. E_2.

Take $\overrightarrow{E^0} := \{[u_1], [u_2], \ldots, [u_n], \ldots\}$ and then $E^0 \in \mathcal{H}^E$. Therefore $H = E^0$ and $G_0 = E_{E^0} = E$. Then by applying source elimination to E, we get E_1. Now take $\overrightarrow{(E_1)^0} := \{[v_1], [v_2], \ldots, [v_n], \ldots\}$ and then $(E_1)^0 \in \mathcal{H}^{E_1}$. Similarly by applying source elimination to E_1, we get E_2. Since $(E_2)^0$ is finite, $L_K(E_2)$ has an identity. Therefore $L_K(E_2)$ has at least one full idempotent and then by Theorem 3.2, $L_K(E)$ has at least one full idempotent.

Example 4.2.

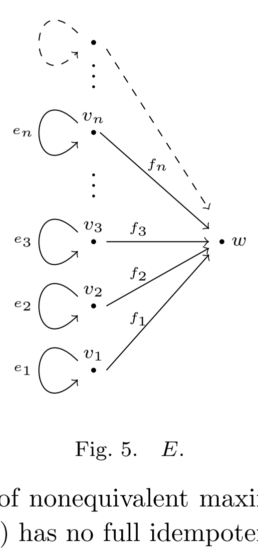

Fig. 5. E.

Since E has infinite number of nonequivalent maximal vertices which are base of loops, by Corollary 3.3 $L_K(E)$ has no full idempotent.

Example 4.3.

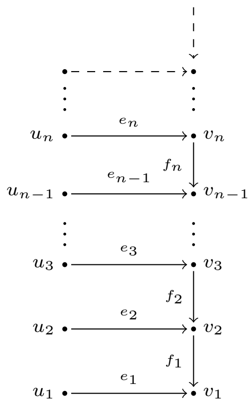

Fig. 6. E.

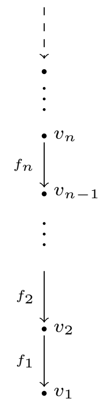

Fig. 7. E_1.

Take $\overrightarrow{E^0} := \{[u_1], [u_2], \ldots, [u_n], \ldots\}$ and then $E^0 \in \mathcal{H}^E$. Therefore $H = E^0$ and $G_0 = E_{E^0} = E$. Then by applying source elimination to E, we get E_1. Now take $H_1 := \{v_1\}$ and then by Lemma 3.2, we have $H_1 \in \mathcal{H}^{E_1}$ and $\overrightarrow{H_1}$ is finite. Then by Theorem 3.1, $L_K(E_1)$ has at least one full idempotent and then by Theorem 3.2, $L_K(E)$ has at least one full idempotent.

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