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Original article

A companion of Ostrowski type inequalities for mappings of bounded variation and some applications

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Abstract

In this paper, we establish a companion of Ostrowski type inequalities for mappings of bounded variation and the quadrature formula is also provided.

Keywords: Bounded variation; Ostrowski type inequalities; Riemann–Stieltjes integral

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1. Introduction

Let $f:[a,b]\to\mathbb{R}$ be a differentiable mapping on (a,b) whose derivative $f':(a,b)\to\mathbb{R}$ is bounded on (a,b), i.e. $\left\|f'\right\|_{\infty}:=\sup_{t\in(a,b)}\left|f'(t)\right|<\infty$. Then, we have the inequality

$$\left|f(x)-\frac{1}{b-a}\int_a^b f(t)dt\right|\le\left[\frac{1}{4}+\frac{\left(x-\frac{a+b}{2}\right)^2}{(b-a)^2}\right](b-a)\left\|f'\right\|_{\infty}, \tag{1.1}$$

for all $x\in[a,b]$ [1]. The constant $\frac{1}{4}$ is the best possible. This inequality is well known in the literature as the *Ostrowski inequality*.

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Definition 1. Let $P : a = x_0 < x_1 < \cdots < x_n = b$ be any partition of $[a, b]$ and let $\Delta f(x_i) = f(x_{i+1}) - f(x_i)$. Then $f(x)$ is said to be of bounded variation if the sum

$$\sum_{i=1}^{m} |\Delta f(x_i)|$$

is bounded for all such partitions. Let f be of bounded variation on $[a, b]$, and $\sum(P)$ denotes the sum $\sum_{i=1}^{n} |\Delta f(x_i)|$ corresponding to the partition P of $[a, b]$. The number

$$\bigvee_a^b (f) := \sup\left\{\sum(P) : P \in P([a, b])\right\},$$

is called the total variation of f on $[a, b]$. Here $P([a, b])$ denote the family of partitions of $[a, b]$.

In [2], Dragomir proved the following Ostrowski type inequalities for functions of bounded variation:

Theorem 1. *Let* $f : [a, b] \to \mathbb{R}$ *be a mapping of bounded variation on* $[a, b]$. *Then*

$$\left|\int_a^b f(t)dt - (b-a) f(x)\right| \leq \left[\frac{1}{2}(b-a) + \left|x - \frac{a+b}{2}\right|\right] \bigvee_a^b (f) \tag{1.2}$$

holds for all $x \in [a, b]$. *The constant* $\frac{1}{2}$ *is the best possible.*

Dragomir gave the following trapezoid inequality in [3]:

Theorem 2. *Let* $f : [a, b] \to \mathbb{R}$ *be a mapping of bounded variation on* $[a, b]$. *Then we have the inequality*

$$\left|\frac{f(a) + f(b)}{2}(b-a) - \int_a^b f(t)dt\right| \leq \frac{1}{2}(b-a) \bigvee_a^b (f). \tag{1.3}$$

The constant $\frac{1}{2}$ *is the best possible.*

We introduce the notation $I_n : a = x_0 < x_1 < \cdots < x_n = b$ for a division of the interval $[a, b]$ with $h_i := x_{i+1} - x_i$ and $v(h) = \max\{h_i : i = 0, 1, \ldots, n-1\}$. Then we have

$$\int_a^b f(t)dt = A_T(f, I_n) + R_T(f, I_n) \tag{1.4}$$

where

$$A_T(f, I_n) := \sum_{i=0}^{n} \frac{f(x_i) + f(x_{i+1})}{2} h_i \tag{1.5}$$

and the remainder term satisfies

$$|R_T(f, I_n)| \leq \frac{1}{2} v(h) \bigvee_a^b (f). \tag{1.6}$$

In [4], Dragomir proved the following companion Ostrowski type inequalities related functions of bounded variation:

Theorem 3. *Assume that the function $f : [a,b] \to R$ is of bounded variation on $[a,b]$. Then we have the inequalities:*

$$\begin{aligned}
&\left|\frac{1}{2}\left[f(x)+f(a+b-x)\right]-\frac{1}{b-a}\int_a^b f(t)dt\right| \\
&\le \frac{1}{b-a}\left[(x-a)\bigvee_a^x(f)+\left(\frac{a+b}{2}-x\right)\bigvee_x^{a+b-x}(f)+(x-a)\bigvee_{a+b-x}^b(f)\right] \\
&\le \begin{cases}
\left[\dfrac{1}{4}+\left|\dfrac{x-\frac{3a+b}{4}}{b-a}\right|\right]\bigvee_a^b(f), \\
\left[2\left(\dfrac{x-a}{b-a}\right)^{\alpha}+\left(\dfrac{\frac{a+b}{2}-x}{b-a}\right)^{\alpha}\right]^{\frac{1}{\alpha}} \\
\times\left[\left[\bigvee_a^x(f)\right]^{\beta}+\left[\bigvee_x^{a+b-x}(f)\right]^{\beta}+\left[\bigvee_{a+b-x}^b(f)\right]^{\beta}\right]^{\frac{1}{\beta}}, & \text{if } \alpha>1,\ \dfrac{1}{\alpha}+\dfrac{1}{\beta}=1, \\
\left[\dfrac{x-a+\frac{b-a}{2}}{b-a}\right]\max\left\{\bigvee_a^x(f),\ \bigvee_x^{a+b-x}(f),\ \bigvee_{a+b-x}^b(f)\right\}
\end{cases}
\end{aligned} \tag{1.7}$$

for any $x \in \left[a, \frac{a+b}{2}\right]$ where $\bigvee_c^d(f)$ denotes the total variation of f on $[c,d]$. The constant $\frac{1}{4}$ is the best possible in the first branch of second inequality in (1.7).

For recent results concerning the above Ostrowski's inequality and other related results see [1–26].

In this work, we obtain a new companion of Ostrowski type integral inequalities for functions of bounded variation. Then we give some applications for our results.

2. Main results

Now, we give a new companion of Ostrowski type integral inequalities for functions of bounded variation:

Theorem 4. *Let $f : [a,b] \to \mathbb{R}$ be a mapping of bounded variation on $[a,b]$. Then, we have the inequality*

$$\begin{aligned}
&\left|\frac{b-a}{4}\left[f(x)+f(a+b-x)+f\left(\frac{a+x}{2}\right)+f\left(\frac{a+2b-x}{2}\right)\right]-\int_a^b f(t)dt\right| \\
&\le \max\left\{\left|x-\frac{3a+b}{4}\right|,\left(\frac{a+b}{2}-x\right),\frac{x-a}{2}\right\}\bigvee_a^b(f)
\end{aligned} \tag{2.1}$$

where $x \in \left[a, \frac{a+b}{2}\right]$ and $\bigvee_c^d(f)$ denotes the total variation of f on $[c,d]$.

Proof. Consider the kernel $P(x,t)$ defined by Qayyum et al. in [7]

$$P(x,t)=\begin{cases}
t-a, & t\in\left[x,\dfrac{a+x}{2}\right] \\
t-\dfrac{3a+b}{4}, & t\in\left(\dfrac{a+x}{2},x\right] \\
t-\dfrac{a+b}{2}, & t\in(x,a+b-x] \\
t-\dfrac{a+3b}{4}, & t\in\left(a+b-x,\dfrac{a+2b-x}{2}\right] \\
t-b, & t\in\left[\dfrac{a+2b-x}{2},b\right].
\end{cases}$$

Integrating by parts, we get

$$\int_a^b P(x,t)df(t)=\frac{b-a}{4}\left[f(x)+f(a+b-x)+f\left(\frac{a+x}{2}\right)+f\left(\frac{a+2b-x}{2}\right)\right]-\int_a^b f(t)dt. \tag{2.2}$$

It is well known that if $g, f : [a,b] \to \mathbb{R}$ are such that g is continuous on $[a,b]$ and f is of bounded variation on $[a,b]$, then $\int_a^b g(t)df(t)$ exists and

$$\left|\int_a^b g(t)df(t)\right| \le \sup_{t\in[a,b]}|g(t)|\bigvee_a^b(f). \tag{2.3}$$

On the other hand, by using (2.3), we get

$$\begin{aligned}
&\left|\int_a^b P(x,t)df(t)\right| \\
&\le \left|\int_a^{\frac{a+x}{2}}(t-a)\,df(t)\right|+\left|\int_{\frac{a+x}{2}}^{x}\left(t-\frac{3a+b}{4}\right)df(t)\right|+\left|\int_x^{a+b-x}\left(t-\frac{a+b}{2}\right)df(t)\right| \\
&\quad+\left|\int_{a+b-x}^{\frac{a+b-x}{2}}\left(t-\frac{a+3b}{4}\right)df(t)\right|+\left|\int_{\frac{a+b-x}{2}}^{b}(t-b)\,df(t)\right| \\
&\le \sup_{t\in\left[a,\frac{a+x}{2}\right]}|t-a|\bigvee_a^{\frac{a+x}{2}}(f)+\sup_{t\in\left[\frac{a+x}{2},x\right]}\left|t-\frac{3a+b}{4}\right|\bigvee_{\frac{a+x}{2}}^{x}(f)+\sup_{t\in[x,a+b-x]}\left|t-\frac{a+b}{2}\right|\bigvee_x^{a+b-x}(f) \\
&\quad+\sup_{t\in\left[a+b-x,\frac{a+2b-x}{2}\right]}\left|t-\frac{a+3b}{4}\right|\bigvee_{a+b-x}^{\frac{a+2b-x}{2}}(f)+\sup_{t\in\left[\frac{a+2b-x}{2},b\right]}|t-b|\bigvee_{\frac{a+2b-x}{2}}^{b}(f) \\
&=\frac{x-a}{2}\bigvee_a^{\frac{a+x}{2}}(f)+\max\left\{\left|x-\frac{3a+b}{4}\right|,\frac{1}{2}\left(\frac{a+b}{2}-x\right)\right\}\bigvee_{\frac{a+x}{2}}^{x}(f)+\left(\frac{a+b}{2}-x\right)\bigvee_x^{a+b-x}(f) \\
&\quad+\max\left\{\left|x-\frac{3a+b}{4}\right|,\frac{1}{2}\left(\frac{a+b}{2}-x\right)\right\}\bigvee_{a+b-x}^{\frac{a+2b-x}{2}}(f)+\frac{x-a}{2}\bigvee_{\frac{a+2b-x}{2}}^{b}(f) \\
&\le \max\left\{\left|x-\frac{3a+b}{4}\right|,\left(\frac{a+b}{2}-x\right),\frac{x-a}{2}\right\}\bigvee_a^b(f).
\end{aligned}$$

This completes the proof. □

Remark 1. If we choose $x=a$ in Theorem 4, the inequality (2.1) reduces the inequality (1.3).

Corollary 1. *Under the assumption of Theorem 4 with $x=\frac{a+b}{2}$, then we have the following inequality*

$$\left|\frac{b-a}{4}\left[2f\left(\frac{a+b}{2}\right)+f\left(\frac{3a+b}{4}\right)+f\left(\frac{a+3b}{4}\right)\right]-\int_a^b f(t)dt\right|\le\frac{1}{4}(b-a)\bigvee_a^b(f). \tag{2.4}$$

The constant $\frac{1}{4}$ is the best possible.

Proof. For proof of the sharpness of the constant, assume that (2.4) holds with a constant $A>0$, that is,

$$\left|\frac{b-a}{4}\left[2f\left(\frac{a+b}{2}\right)+f\left(\frac{3a+b}{4}\right)+f\left(\frac{a+3b}{4}\right)\right]-\int_a^b f(t)dt\right|\le A(b-a)\bigvee_a^b(f). \tag{2.5}$$

If we choose $f : [a, b] \to \mathbb{R}$ with

$$f(x) = \begin{cases} 1, & \text{if } x \in \left\{ \dfrac{a+b}{2}, \dfrac{3a+b}{4}, \dfrac{a+3b}{4} \right\} \\ 0, & \text{if } x \in [a, b] \,/ \left\{ \dfrac{a+b}{2}, \dfrac{3a+b}{4}, \dfrac{a+3b}{4} \right\} \end{cases}$$

then f is of bounded variation on $[a, b]$, and

$$2f\left(\frac{a+b}{2}\right) + f\left(\frac{3a+b}{4}\right) + f\left(\frac{a+3b}{4}\right) = 4, \quad \int_a^b f(t)dt = 0, \quad \text{and} \quad \bigvee_a^b (f) = 4,$$

giving in (2.5), $1 \leq 4A$, thus $A \geq \frac{1}{4}$. ☐

Corollary 2. *Under the assumption of Theorem 4 with* $x = \frac{3a+b}{4}$, *then we get the inequality*

$$\left| \frac{b-a}{4} \left[f\left(\frac{3a+b}{4}\right) + f\left(\frac{a+3b}{4}\right) + f\left(\frac{7a+b}{8}\right) + f\left(\frac{a+7b}{8}\right) \right] - \int_a^b f(t)dt \right| \\ \leq \frac{1}{8}(b-a) \bigvee_a^b (f). \tag{2.6}$$

The constant $\frac{1}{8}$ *is the best possible.*

Proof. For proof of the sharpness of the constant, assume that (3.4) holds with a constant $B > 0$, that is,

$$\left| \frac{b-a}{4} \left[f\left(\frac{3a+b}{4}\right) + f\left(\frac{a+3b}{4}\right) + f\left(\frac{7a+b}{8}\right) + f\left(\frac{a+7b}{8}\right) \right] - \int_a^b f(t)dt \right| \\ \leq B(b-a) \bigvee_a^b (f). \tag{2.7}$$

If we choose $f : [a, b] \to \mathbb{R}$ with

$$f(x) = \begin{cases} 1, & \text{if } x \in \left\{ \dfrac{3a+b}{4}, \dfrac{a+3b}{4}, \dfrac{7a+b}{8}, \dfrac{a+7b}{8} \right\} \\ 0, & \text{if } x \in [a, b] \,/ \left\{ \dfrac{3a+b}{4}, \dfrac{a+3b}{4}, \dfrac{7a+b}{8}, \dfrac{a+7b}{8} \right\} \end{cases}$$

then f is of bounded variation on $[a, b]$, and

$$f\left(\frac{3a+b}{4}\right) + f\left(\frac{a+3b}{4}\right) + f\left(\frac{7a+b}{8}\right) + f\left(\frac{a+7b}{8}\right) = 4,$$

$$\int_a^b f(t)dt = 0, \quad \text{and} \quad \bigvee_a^b (f) = 8,$$

giving in (2.7), $1 \leq 8B$, thus $B \geq \frac{1}{8}$. ☐

Corollary 3. *Let f be defined as in Theorem 4, and, additionally, if* $f(x) = f(a+b-x)$, *then we have*

$$\left| \frac{b-a}{4} \left[2f(x) + f\left(\frac{a+x}{2}\right) + f\left(\frac{a+2b-x}{2}\right) \right] - \int_a^b f(t)dt \right| \\ \leq \max\left\{ \left| x - \frac{3a+b}{4} \right|, \left(\frac{a+b}{2} - x \right), \frac{x-a}{2} \right\} \bigvee_a^b (f). \tag{2.8}$$

Corollary 4. *If we choose $x = a$ in Corollary 3, then we have the inequality*

$$\left| \frac{3f(a)+f(b)}{4}(b-a) - \int_a^b f(t)dt \right| \le \frac{1}{2}(b-a)\bigvee_a^b(f).$$

The constant $\frac{1}{2}$ is the best possible.

The sharpness of the constant can be proved similarly Corollaries 1 and 2, so it is omitted.

Corollary 5. *Under the assumption of Theorem 4, suppose that $f \in C^1[a,b]$. Then we have*

$$\begin{aligned}&\left| \frac{b-a}{4}\left[f(x) + f(a+b-x) + f\left(\frac{a+x}{2}\right) + f\left(\frac{a+2b-x}{2}\right)\right] - \int_a^b f(t)dt \right| \\ &\quad \le \max\left\{ \left| x - \frac{3a+b}{4}\right|, \left(\frac{a+b}{2} - x\right), \frac{x-a}{2} \right\} \|f'\|_1\end{aligned}$$

for all $x \in \left[a, \frac{a+b}{2}\right]$. Here as subsequently $\|.\|_1$ is the L_1-norm

$$\|f'\|_1 := \int_a^b f'(t)dt.$$

Corollary 6. *Under the assumption of Theorem 4, let $f : [a,b] \to \mathbb{R}$ be a Lipschitzian with the constant $L > 0$. Then*

$$\begin{aligned}&\left| \frac{b-a}{4}\left[f(x) + f(a+b-x) + f\left(\frac{a+x}{2}\right) + f\left(\frac{a+2b-x}{2}\right)\right] - \int_a^b f(t)dt \right| \\ &\quad \le \max\left\{ \left| x - \frac{3a+b}{4}\right|, \left(\frac{a+b}{2} - x\right), \frac{x-a}{2} \right\} (b-a)L\end{aligned}$$

for all $x \in \left[a, \frac{a+b}{2}\right]$.

3. Application to quadrature formula

We now introduce the intermediate points $\xi_i \in \left[x_i, \frac{x_i+x_{i+1}}{2}\right]$ $(i = 0, 1, \ldots, n-1)$ in the division $I_n : a = x_0 < x_1 < \cdots < x_n = b$. Let $h_i := x_{i+1} - x_i$ and $v(h) = \max\{h_i : i = 0, 1, \ldots, n-1\}$ and define the sum

$$A(f, I_n, \xi) := \frac{1}{4}\sum_{i=0}^{n} h_i \left[f(\xi_i) + f(x_i + x_{i+1} - \xi_i) + f\left(\frac{x_i+\xi_i}{2}\right) + f\left(\frac{x_i + 2x_{i+1} - \xi_i}{2}\right)\right]. \tag{3.1}$$

Then the following theorem holds:

Theorem 5. *Let f be as Theorem 4. Then*

$$\int_a^b f(t)dt = A(f, I_n, \xi) + R(f, I_n, \xi) \tag{3.2}$$

where $A(f, I_n, \xi)$ is defined as above and the remainder term $R(f, I_n, \xi)$ satisfies

$$|R(f, I_n, \xi)| \le \max_{i\in\{0,1,\ldots,n-1\}}\left[\max\left\{ \left| \xi_i - \frac{3x_i + x_{i+1}}{4}\right|, \left(\frac{x_i + x_{i+1}}{2} - \xi_i\right), \frac{\xi_i - x_i}{2}\right\}\right] \bigvee_a^b(f). \tag{3.3}$$

Proof. Applying Theorem 4 to the interval $[x_i, x_{i+1}]$ $(i = 0, 1, \ldots, n-1)$, we have

$$\begin{aligned}&\left| \frac{h_i}{4}\left[f(\xi_i) + f(x_i + x_{i+1} - \xi_i) + f\left(\frac{x_i+\xi_i}{2}\right) + f\left(\frac{x_i + 2x_{i+1} - \xi_i}{2}\right)\right] - \int_{x_i}^{x_{i+1}} f(t)dt \right| \\ &\quad \le \max\left\{ \left| \xi_i - \frac{3x_i + x_{i+1}}{4}\right|, \left(\frac{x_i + x_{i+1}}{2} - \xi_i\right), \frac{\xi_i - x_i}{2}\right\} \bigvee_{x_i}^{x_{i+1}}(f)\end{aligned} \tag{3.4}$$

for all $i \in \{0, 1, \ldots, n-1\}$. Summing the inequality (3.4) over i from 0 to $n-1$ and using the generalized triangle inequality, we have

$$\begin{aligned} |R(f, I_n, \xi)| &\le \sum_{i=0}^{n} \max\left\{\left|\xi_i - \frac{3x_i + x_{i+1}}{4}\right|, \left(\frac{x_i + x_{i+1}}{2} - \xi_i\right), \frac{\xi_i - x_i}{2}\right\} \bigvee_{x_i}^{x_{i+1}}(f) \\ &\le \max_{i\in\{0,1,\ldots,n-1\}}\left[\max\left\{\left|\xi_i - \frac{3x_i + x_{i+1}}{4}\right|, \left(\frac{x_i + x_{i+1}}{2} - \xi_i\right), \frac{\xi_i - x_i}{2}\right\}\right] \sum_{i=0}^{n} \bigvee_{x_i}^{x_{i+1}}(f) \\ &= \max_{i\in\{0,1,\ldots,n-1\}}\left[\max\left\{\left|\xi_i - \frac{3x_i + x_{i+1}}{4}\right|, \left(\frac{x_i + x_{i+1}}{2} - \xi_i\right), \frac{\xi_i - x_i}{2}\right\}\right] \bigvee_{a}^{b}(f) \end{aligned}$$

which completes the proof. □

Remark 2. If we choose $\xi_i = x_i$ in Theorem 5, we get (1.4) with (1.5) and (1.6).

Corollary 7. *If we choose* $\xi_i = \frac{x_i + x_{i+1}}{2}$ *in Theorem 5, then we have*

$$\int_a^b f(t)dt = A(f, I_n) + R(f, I_n)$$

where

$$A(f, I_n) := \frac{1}{4}\sum_{i=0}^{n} h_i \left[2f\left(\frac{x_i + x_{i+1}}{2}\right) + f\left(\frac{3x_i + x_{i+1}}{2}\right) + f\left(\frac{x_i + 3x_{i+1}}{2}\right)\right]$$

and the remainder term $R(f, I_n)$ *satisfies*

$$|R(f, I_n)| \le \frac{1}{4}\upsilon(h) \bigvee_a^b(f).$$

Corollary 8. *If we choose* $\xi_i = \frac{3x_i + x_{i+1}}{4}$ *in Theorem 5, then we have*

$$\int_a^b f(t)dt = A(f, I_n) + R(f, I_n)$$

where

$$A(f, I_n) := \frac{1}{4}\sum_{i=0}^{n} h_i \left[f\left(\frac{3x_i + x_{i+1}}{2}\right) + f\left(\frac{x_i + 3x_{i+1}}{2}\right) + f\left(\frac{7x_i + x_{i+1}}{8}\right) + f\left(\frac{x_i + 7x_{i+1}}{8}\right)\right]$$

and the remainder term $R(f, I_n)$ *satisfies*

$$|R(f, I_n)| \le \frac{1}{8}\upsilon(h) \bigvee_a^b(f).$$

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