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Design, modelling and simulation of a new nonlinear and full adaptive backstepping speed tracking controller for uncertain PMSM

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ABSTRACT

In this study, a new nonlinear and full adaptive backstepping speed tracking control scheme is developed for an uncertain permanent magnet synchronous motor (PMSM). Except for the number of pole pairs, all the other parameters in both PMSM and load dynamics are assumed unknown. Three phase currents and rotor speed are supposed to be measurable and available for feedback in the controller design. By designing virtual control inputs and choosing appropriate Lyapunov functions, the final control and parameter estimation laws are derived. The overall control system possesses global asymptotic stability; all the signals in the closed loop system remain bounded, according to stability analysis results based on Lyapunov stability theory. Further, the proposed controller does not require computation of regression matrices, with the result that take the nonlinearities in quite general. Simulation results clearly exhibit that the controller guarantees tracking of a time varying desired reference speed trajectory under all the uncertainties in both PMSM and load dynamics without singularity and overparameterization. The results also show that all the parameter estimates converge to their true values on account of the fact that reference speed signal chosen to be sufficiently rich ensures persistency of excitation condition. Consequently, the proposed controller ensures strong robustness against all the parameter uncertainties and unknown bounded load torque disturbance in the PMSM drive system. Numerical simulations demonstrate the performance and feasibility of the proposed controller.

1. Introduction

PMSM has been attracting more and more attention in high performance electric drive applications since it has certain superiorities such as; high efficiency, high power factor, superior power density, large torque to inertia ratio and long life over other kinds of motors such as DC motors and induction motors [1]. However, PMSM drive systems have the nonlinear dynamics containing parameter uncertainties and unknown external disturbances. In order to meet high performance requirements of PMSM industrial drive applications, robust or adaptive or other control schemes dealing with parameter uncertainties and unknown external disturbances have widely been studied thus far.

The sliding mode variable structure control (SMC) approach has been extensively studied owing to its robustness against model uncertainties, parameter perturbations and external disturbances [2]. The worst drawback of it is the chattering problem around the steady state, which restricts its application. A robust velocity control of PMSM using boundary layer integral sliding mode control technique was proposed in [3] for the purpose of reducing the chattering phenomenon, however;

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causes the steady state error [4]. When the controlled system is under the influence of matched perturbations, conventional SMC ensures asymptotic stability [5,6]. As for mismatched perturbations, it is necessary to design a specific sliding surface so as to achieve asymptotic stability. Hence, some researchers designed novel sliding surface functions for mismatched perturbed systems [7]. The control scheme guarantees asymptotic stability and does not need the knowledge of the upper bounds of the perturbations. Nevertheless, designing a specific sliding function is a difficult task.

The control methods based on artificial intelligence [8–10] have a good robustness despite parameter variations and unknown external disturbances since its design is independent of mathematical model of the plant. Gupta et al. presented a detailed review study about the artificial intelligence based control of PMSMs and pointed out that the fuzzy logic and neural network control are most mature and attractive methods for the PMSM drive systems [11]. Nonetheless, real time implementation of these and suchlike methods is difficult thanks to their heavy computational effort and algorithm complexity.

Yucelen et al. studied state dependent Riccati equation (SDRE) based controller, which is also referred to nonlinear quadratic optimal control [12]. The controller requires solving an algebraic Riccati equation [13,14], hence; has relatively complex formulation and requires heavy computational effort.

Model reference adaptive control (MRAC), which is often used for estimating state variables of nonlinear control systems during periods of time when the measurements of the related state variables are not available for feedback [15], based nonlinear speed control of interior PMSM was proposed in [16]. The controller was developed in the sense of the input to output feedback linearization scheme, the utilization of which may cause to cancel some useful nonlinearity out [17]. The partial plant uncertainties; load torque disturbance and permanent magnet flux, are taken into account and this in turn results in that the exact values of the other plant parameters are required to be known in the controller design. As another consequence of that, the control system gives unsatisfactory performance in the event that these known parameters vary because of the different operating conditions such as variation of temperature, saturation and external disturbance etc.

Backstepping control is a new type recursive and systematic design methodology for the feedback control of uncertain nonlinear systems, particularly for the systems with matched uncertainties [18,19]. This method removes the difficulties in obtaining a control Lyapunov function with the help of a number of recursive steps that never exceed the system order. Every step produces a virtual control variable making the original high order system simple; thus, the final control outputs can be derived systematically through suitable Lyapunov functions. An adaptive robust nonlinear controller straightforwardly derived using this control method is proposed for the speed control of PMSM [20]. The controller is robust against stator resistance, viscous friction uncertainties and load torque disturbance. However, this approach uses the feedback linearization, which its main drawback mentioned above. Other adaptive nonlinear backstepping design methods that do not use the linearization theorems are proposed for the control of electromechanical systems [17,21–29]. Even though adaptive backstepping control schemes above satisfy high performance requirements for industrial electric drive applications, there are still some important drawbacks whose characteristics differ in respect to the methodology used in the controller design. It is worthy to mention at this point that these drawbacks are regarded as inconvenient in terms of real time implementation. We can arrange them all in the following order:

(**D1**) The design complexity of the controller highly increases owing to the regression matrices under the assumption of linear-in-the-parameters.

(**D2**) The fact that partial plant uncertainties are taken into account requires the exact values of the other plant parameters to be known in the controller design. Hence, the resulting control scheme becomes sensitive to the variations, due to different operating conditions such as temperature, saturation, external disturbances and so on, of these parameters assumed known.

(**D3**) The linearization theorems used in the controller design may cause to cancel some useful nonlinearity.

(**D4**) In some control systems, the final time derivative of the Lyapunov function includes the positive definite terms which cover partial model knowledge of plant. It is possible for these positive terms to cause the derivative to be positive definite, and in this case the controller cannot preserve asymptotic stability. In order to ensure the asymptotic stability of the control system, it is mandatory to properly select feedback gains to satisfy that the final time derivative of the Lyapunov function is nonpositive. This case requires estimating possible values of the upper or/and lower bounds of the plant parameters which the positive definite terms comprise. As stated in Section 2, there is mostly no way to measure or calculate the exact values and the lower–upper bounds of the plant parameters that vary with operating conditions. Suchlike control schemes do not ensure the asymptotic stability of the control system, when the perturbations of these parameters go beyond the estimated upper or/and lower bounds.

(**D5**) The singularity occurs if any parameter estimation term appears as a denominator of any control input. Then, the related control inputs become quite large if these estimation terms tend to zero. As to the overparameterization, this means that the number of adaptive parameters is more than the number of unknown plant parameters. In this case, selecting adaptation gains becomes a more difficult task.

In the previous paper [29], we developed a nonlinear and adaptive backstepping controller under five parameter uncertainties, which other two parameters, the number of pole pairs and permanent magnet flux, were assumed known. Furthermore, the asymptotic stability of the control system depended on that k_1 and k_2 feedback gains satisfy a certain inequality. As a result, the controller proposed in [29] suffers from the (D2) and (D4).

The novelty of this study, as an extension of [29], is to relax the drawbacks of the controller proposed in [29] while preserving its all advantages. In this context, we propose a new nonlinear and full adaptive backstepping speed tracking controller for an uncertain PMSM, which has a full adaptive structure and takes six parameter uncertainties into account. As in addition to [29], the permanent magnet flux of PMSM is also considered uncertain and hence the (D2) is eliminated. The number of pole pairs is not estimated adaptively since it is nameplate information and does not vary with different operating conditions. Moreover, the asymptotic stability of the proposed controller does not depend on any inequality that the feedback gains need to meet, which eliminates the (D4). The contributions related to overcoming the (D2) and (D4) are summarized in the item ii. and iv. in the section of the concluding remarks.

Stability results, adaptation and control laws are derived using appropriate Lyapunov functions. The asymptotic stability of the resulting closed loop system is guaranteed in the sense of Lyapunov stability theorem. The resulted control scheme can track a time varying reference signal quite correctly under all the parameter and disturbance uncertainties in PMSM drive system. The paper is structured as follows: The dynamics of PMSM with the uncertainties is introduced in the second section. The third section reveals the proposed control design scheme in connection with Lyapunov stability theorem. The fourth section covers the simulations results and discussion about the results. The last section concludes the paper with the contributions obtained by the proposed controller.

2. Dynamics of PMSM with the uncertainties

The mathematical model of a conventional surface mounted PMSM can be given with standard assumptions in the *d-q* frame [21,30,31] in the following:

$$\frac{d\omega}{dt} = \frac{3P\lambda_m}{2J} i_q - \frac{B}{J}\omega - \frac{T_L}{J}, \quad (1)$$

$$\frac{di_q}{dt} = -\frac{R}{L} i_q - \omega_e i_d - \frac{\lambda_m}{L}\omega_e + \frac{1}{L}V_q; (\omega_e = P\omega), \quad (2)$$

$$\frac{di_d}{dt} = -\frac{R}{L} i_d + \omega_e i_q + \frac{1}{L}V_d, \quad (3)$$

where i_d and i_q are the *d-q* axis currents, V_d and V_q are the *d-q* axis voltages, R and L denotes the stator resistance and inductance per phase respectively, P is the number of pole pairs, ω is the rotor speed, λ_m is the permanent magnet flux, J is the rotor moment of inertia, B is the viscous friction factor and T_L also represents the applied load torque disturbance. It follows from the equations above that PMSM is highly nonlinear system owing to the cross coupling effect between the electrical current and speed state equations. In a practical way, measurement and calculation of the electrical parameters R, L and λ_m are available; however, it must be noted that they vary with operating conditions; primarily temperature and saturation effects. As for the mechanical parameters J and B, it is not possible to practically measure or calculate the exact values of them. These parameters vary with operating conditions as well, primarily applied load torque disturbance. Even worse, the load torque disturbance is always unknown. In that respect, it is clear that industrial PMSM drive systems encounter unavoidable parameter variations and immeasurable disturbances.

It is explicitly apparent from the nonlinear and adaptive control perspective that PMSMs, as all electromechanical systems, have nonlinear dynamics that is subject to parameter uncertainties and unknown external disturbances. For this reason, it is necessary to take all the nonlinearities into account and adaptively estimate all the uncertainties in PMSM drive system in order to achieve precise control performance. The next section covers the proposed controller scheme coping with all the uncertainties in both PMSM and load dynamics.

3. Controller design methodology

The main control objective is to keep all the signals in the closed loop system bounded and ensures global asymptotic convergence of the speed and current tracking errors to zero despite all the uncertainties in both PMSM and load dynamics. The objective can only be reached by making the design of the control inputs independent of all the parameters of PMSM and unknown load torque disturbance. Thus, there is the need for online estimation of all the uncertainties adaptively.

3.1. The proposed nonlinear and full adaptive backstepping controller design

In the backstepping procedure, a virtual control state is firstly defined and then it is forced to become a stabilizing function, which a corresponding error variable is generated. Consequently, by appropriate designing the related control input by virtue of Lyapunov stability theory, the error variable can be stabilized. For convenience, we will firstly redefine the motor model with the parameter and disturbance uncertainties. The parametric transformations introduced to the mathematical model of PMSM in (1)–(3) are given in the following:

$$a_1 = \frac{2B}{3\lambda_m}, \quad a_2 = \frac{2T_L}{3\lambda_m}, \quad a_3 = \frac{2J}{3\lambda_m}, \quad b_1 = R, \ b_2 = L, \ b_3 = \lambda_m. \quad (4)$$

The mathematical model of PMSM according as the new parameters can be redefined as:

$$\begin{aligned} &\frac{\mathbf{1}}{P}a_3\frac{d\omega}{dt} = i_q - \frac{\mathbf{1}}{P}(a_1\omega - a_2),\\ &b_2\frac{di_q}{dt} = -b_{\mathbf{1}}i_q - b_2\omega_e i_d - b_3\omega_e + V_q,\\ &b_2\frac{di_d}{dt} = -b_{\mathbf{1}}i_d + b_2\omega_e i_q + V_d. \end{aligned} \tag{5}$$

Suppose that all the parameters above are unknown but positive finite constants, then:

$$\tilde{x}_i = x_i - \hat{x}_i; \quad \dot{\tilde{x}} = -\dot{\hat{x}}_i, \quad x = (a,b),\ i = (1,2,3). \tag{6}$$

The definitions with cap indicate the parameter estimations and with tilde indicate their estimation errors individually in (6). Taking all above into consideration, the motor model in terms of the parameter estimations and estimation errors is transformed into the following form:

$$\begin{aligned} &\frac{\mathbf{1}}{P}a_3\frac{d\omega}{dt} = i_q - \frac{\mathbf{1}}{P}(\tilde{a}_{\mathbf{1}}\omega - \hat{a}_{\mathbf{1}}\omega - \tilde{a}_2 - \hat{a}_2),\\ &b_2\frac{di_q}{dt} = -\tilde{b}_1 i_q - \hat{b}_1 i_q - \tilde{b}_2\omega_e i_d - \hat{b}_2\omega_e i_d - \tilde{b}_3\omega_e - \hat{b}_3\omega_e + V_q,\\ &b_2\frac{di_d}{dt} = -\tilde{b}_1 i_d - \hat{b}_1 i_d + \tilde{b}_2\omega_e i_q + \hat{b}_2\omega_e i_q + V_d, \end{aligned} \tag{7}$$

where the number of pole pairs P is the only one parameter assumed known since it is nameplate information and does not vary with different operating conditions. With the help of this type of parametric transformation, the singularity and overparameterization problems are avoided, which can be clearly seen at the controller design stage. Taking all the dynamics of the PMSM drive system into account, the following commonly assumptions are made as:

(**A1**) All the state variables i_q, i_d and ω are available for feedback.

(**A2**) The signal $\omega_d \in \mathbb{R}$ is the desired reference speed trajectory, differentiable in a numerical way and bounded with the derivatives $\omega_d, \dot{\omega}_d, \ddot{\omega}_d \in L_\infty$.

(**A3**) The load torque disturbance and viscous friction factor vary within the known bounds. $B = B_N(1+\Delta_B)$; $T_L = T_{LN}(1+\Delta_T); \Delta_i^{\min} \leqslant \Delta_i \leqslant \Delta_i^{\max}, i = (B,T)$
where B_N and T_{LN} indicate the relevant nominal values, Δ_B and Δ_T are the unknown disturbances which is bounded by the known $\Delta_i^{\min}$ and $\Delta_i^{\max}$ intervals.

(**A4**) The total effect of the disturbances, which is called the lumped disturbance consisting of the viscous friction and load torque disturbance, on the control system varies within the known bounds. $d = B\omega + T_L; d = d_N(1+\Delta_d)$; $\Delta_d^{\min} \leqslant \Delta_d \leqslant \Delta_d^{\max}$

where d_N implies the nominal value, Δ_d is the unknown lumped disturbance which is bounded by the known $\Delta_d^{\min}$ and $\Delta_d^{\max}$ intervals.

Then, the overall control design can be established by three steps in the following order:

Step 1: The speed tracking errors can be defined as:

$$e = \omega - \omega_d, \tag{8}$$

where ω is the actual rotor speed. In order to stabilize the speed tracking error dynamics, the first positive definite Lyapunov function can be defined as follows:

$$V_1 = \frac{1}{2P}a_3 e^2. \tag{9}$$

Time derivative of (9) is given, after some mathematical manipulation, in the following:

$$\dot{V}_1 = e\left[i_q - \frac{\mathbf{1}}{P}(\tilde{a}_1\omega - \hat{a}_1\omega - \tilde{a}_2 - \hat{a}_2 - \tilde{a}_3\dot{\omega}_d - \hat{a}_3\dot{\omega}_d)\right]. \tag{10}$$

For the purpose of applying backstepping procedure to (10), the term $\pm i_{qdes}$ is added and subtracted to right hand side of (10):

$$\dot{V}_1 = e\left[i_q - \frac{\mathbf{1}}{P}(\tilde{a}_1\omega - \hat{a}_1\omega - \tilde{a}_2 - \hat{a}_2 - \tilde{a}_3\dot{\omega}_d - \hat{a}_3\dot{\omega}_d) \pm i_{qdes}\right]. \tag{11}$$

At this point, the current tracking errors that backstepping will be applied on are defined as follows:

$$e_q = i_q - i_{qdes}, \quad e_d = i_d - i_{d\_d}, \tag{12}$$

where i_{d_d} and i_{qdes} describe the desired trajectories of the three phase currents on d and q axes respectively. By taking these desired trajectories from (12) and inserting the resulting equation into (11) yield:

$$\dot{V}_1 = e\left[e_q + i_{qdes} - \frac{1}{P}(\tilde{a}_1\omega - \hat{a}_1\omega - \tilde{a}_2 - \hat{a}_2 - \tilde{a}_3\dot{\omega}_d - \hat{a}_3\dot{\omega}_d)\right]. \tag{13}$$

In order to stabilize the speed tracking error dynamics, that is, to guarantee rotor speed ω tracks the desired reference speed trajectory ω_d,i_{d_d} and i_{qdes} can be designed using the backstepping and vector control concepts respectively as:

$$i_{qdes} = \frac{1}{P}(\hat{a}_1\omega + \hat{a}_2 + \hat{a}_3\dot{\omega}_d) - k_1 e, \quad i_{d\_d} = \mathbf{0}, \tag{14}$$

where k_1 is the feedback gain. Therefore, we obtain the derivative of the first Lyapunov function in terms of the parameter estimations and estimation errors by substituting (14) into (13) as follows:

$$\dot{V}_1 = e\left[e_q - \frac{1}{P}(\tilde{a}_1\omega - \tilde{a}_2 - \tilde{a}_3\dot{\omega}_d) - k_1 e\right]. \tag{15}$$

Step 2: Now, backstepping can be applied on e_q by defining the second positive definite Lyapunov function to stabilize q axis current tracking error dynamics as:

$$V_2 = \frac{1}{2} b_2 e_q^2 \tag{16}$$

(**A5**) The time derivative of the desired q axis current i_{qdes} is differentiable in a numerical way, $\Delta i_{qdes}/\Delta t$.

The time derivative of the desired q axis current in the second Lyapunov function of (16) is expressed under the assumption (5) as the following:

$$\dot{V}_2 = e_q\left[-\tilde{b}_1 i_q - \hat{b}_1 i_q - \tilde{b}_2\omega_e i_d - \hat{b}_2\omega_e i_d - \tilde{b}_3\omega_e - \hat{b}_3\omega_e + V_q - \tilde{b}_2\frac{\Delta i_{qdes}}{\Delta t} - \hat{b}_2\frac{\Delta i_{qdes}}{\Delta t}\right]. \tag{17}$$

The first control input can be designed to stabilize q axis current tracking error dynamics as:

$$V_q = \hat{b}_1 i_q + \hat{b}_2\omega_e i_d + \hat{b}_3\omega_e + \hat{b}_2\frac{\Delta i_{qdes}}{\Delta t} - k_2 e_q - e \tag{18}$$

where k_2 is the feedback gain. Substituting (18) into (17) yields:

$$\dot{V}_2 = e_q\left[-\tilde{b}_1 i_q - \tilde{b}_2\omega_e i_d - \tilde{b}_3\omega_e - \tilde{b}_2\frac{\Delta i_{qdes}}{\Delta t} - k_2 e_q - e\right]. \tag{19}$$

At this step, the third positive definite Lyapunov function can be defined to stabilize d axis current tracking error dynamics as follows:

$$V_3 = \frac{1}{2} b_2 e_d^2, \tag{20}$$

whose time derivative becomes:

$$\dot{V}_3 = e_d[-\tilde{b}_1 i_d - \hat{b}_1 i_d + \tilde{b}_2\omega_e i_q + \hat{b}_2\omega_e i_q + V_d], \tag{21}$$

V_d can directly be used to stabilize d axis current tracking error dynamics as:

$$V_d = \hat{b}_1 i_d + \hat{b}_2\omega_e i_q - k_3 e_d, \tag{22}$$

where k_3 is the feedback gain. Inserting (22) into (21) renders the time derivative of (20):

$$\dot{V}_3 = e_d[-\tilde{b}_1 i_d + \tilde{b}_2\omega_e i_q - k_3 e_d]. \tag{23}$$

Step 3: The last positive definite Lyapunov function can be described for the overall control system in order to determine the parameter adaptation laws and then stability of the control system. Our goal at this step is to render time derivative of the Lyapunov function nonpositive.

$$V = V_1 + V_2 + V_3 + \frac{1}{2\theta_1}\tilde{a}_1^2 + \frac{1}{2\theta_2}\tilde{a}_2^2 + \frac{1}{2\theta_3}\tilde{a}_3^2 + \frac{1}{2\theta_4}\tilde{b}_1^2 + \frac{1}{2\theta_5}\tilde{b}_2^2 + \frac{1}{2\theta_6}\tilde{b}_3^2. \tag{24}$$

Taking the time derivative of the Lyapunov function and inserting 23, 19 and 15 into the resulting equation yield:

$$\begin{aligned}\dot{V} = {} & e\left[e_q - \frac{1}{P}(\tilde{a}_1\omega - \tilde{a}_2 - \tilde{a}_3\dot{\omega}_d) - k_1 e\right] + e_q\left[-\tilde{b}_1 i_q - \tilde{b}_2\omega_e i_d - \tilde{b}_3\omega_e - \tilde{b}_2\frac{\Delta i_{qdes}}{\Delta t} - k_2 e_q\right] + e_d[-\tilde{b}_1 i_d + \tilde{b}_2\omega_e i_q \\ & - k_3 e_d] + \frac{1}{\theta_1}\tilde{a}_1\dot{\tilde{a}}_1 + \frac{1}{\theta_2}\tilde{a}_2\dot{\tilde{a}}_2 + \frac{1}{\theta_3}\tilde{a}_3\dot{\tilde{a}}_3 + \frac{1}{\theta_4}\dot{\tilde{b}}_1\tilde{b}_1 + \frac{1}{\theta_5}\tilde{b}_2\dot{\tilde{b}}_2 + \frac{1}{\theta_6}\tilde{b}_3\dot{\tilde{b}}_3,\end{aligned} \tag{25}$$

where θ_1, θ_2, θ_3, θ_4, θ_5 and θ_6 are the positive finite adaptation gains. The derivative can clearly be expressed again with certain mathematical manipulations as:

$$\begin{aligned}\dot{V} = {} & -k_1 e^2 - k_2 e_q^2 - k_3 e_d^2 + \tilde{a}_1\left[-\frac{1}{\theta_1}\dot{\hat{a}}_1 - \frac{1}{P}e\omega\right] + \tilde{a}_2\left[-\frac{1}{\theta_2}\dot{\hat{a}}_2 - \frac{1}{P}e\right] + \tilde{a}_3\left[-\frac{1}{\theta_3}\dot{\hat{a}}_3 - \frac{1}{P}e\dot{\omega}_d\right] \\ & + \tilde{b}_1\left[-\frac{1}{\theta_4}\dot{\hat{b}}_1 - i_q e_q - i_d e_d\right] + \tilde{b}_2\left[-\frac{1}{\theta_5}\dot{\hat{b}}_2 - \omega_e i_d e_q + \omega_e i_q e_d - \frac{\Delta i_{qdes}}{\Delta t}e_q\right] + \tilde{b}_3\left[-\frac{1}{\theta_6}\dot{\hat{b}}_3 - \omega_e e_q\right].\end{aligned} \tag{26}$$

So as to ensure asymptotic stability of the overall control system, the time derivative of the Lyapunov function in (26) can be rendered nonpositive by choosing the update laws:

$$\begin{aligned}&\text{(a) } \dot{\hat{a}}_1 = -\theta_1\frac{1}{P}e\omega \quad \text{(b) } \dot{\hat{a}}_2 = -\theta_2\frac{1}{P}e \quad \text{(c) } \dot{\hat{a}}_3 = -\theta_3\frac{1}{P}e\dot{\omega}_d, \\ &\text{(d) } \dot{\hat{b}}_1 = -\theta_4(i_q e_q + i_d e_d) \quad \text{(e) } \dot{\hat{b}}_2 = -\theta_5\begin{pmatrix}\omega_e i_d e_q - \omega_e i_q e_d \\ +\frac{\Delta i_{qdes}}{\Delta t}e_q\end{pmatrix} \quad \text{(f) } \dot{\hat{b}}_3 = -\theta_6\omega_e e_q.\end{aligned} \tag{27}$$

Substituting the update laws of (27a–f) in (26) results in:

$$\dot{V} = -k_1 e^2 - k_2 e_q^2 - k_3 e_d^2 \leqslant 0, \tag{28}$$

which means that there exists a nonpositive, or negative semi definite, time derivative of the Lyapunov function $\dot{V} \leqslant 0$ outside the equilibrium point (0,0,0) in the (e, e_d, e_q) coordinates.

3.2. Stability analysis

Theorem 3.1. *Invoking LaSalle's invariance set theorem, it is evident that* $V(e, e_d, e_q, \tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{b}_1, \tilde{b}_2, \tilde{b}_3) : R^9 \times R_+ \to R_+$ *is continuously differentiable scalar function of the states, positive definite and radially unbounded such that* $\dot{V}(e, e_d, e_q) \leqslant 0, \forall(e, e_d, e_q, \tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{b}_1, \tilde{b}_2, \tilde{b}_3) \in R^9$. *Then, ninth dimensional state converge to the largest invariant set M of (15), (19), (23) and (27) contained in the set* $E = \{(e, e_d, e_q, \tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{b}_1, \tilde{b}_2, \tilde{b}_3) \in R^9 | (e, e_d, e_q) = (0,0,0)\}$ *where* $\dot{V} = 0$, *and there is no solution that can stay forever in E except for* $(e, e_d, e_q) \equiv (0,0,0)$. *On this invariant set M, we have* $(e, e_d, e_q, \dot{e}, \dot{e}_d, \dot{e}_q) = (0,0,0,0,0,0)$. *Setting* $(e, e_d, e_q) = (0,0,0)$ *in (27), we obtain* $(\dot{\hat{a}}_1, \dot{\hat{a}}_2, \dot{\hat{a}}_3, \dot{\hat{b}}_1, \dot{\hat{b}}_2, \dot{\hat{b}}_3) = (0,0,0,0,0,0)$. *Setting* $(\dot{e}, \dot{e}_d, \dot{e}_q) = (0,0,0)$ *in (15), (19) and (23), we have* $0 = \tilde{a}_1\omega - \tilde{a}_2 - \tilde{a}_3\dot{\omega}_d, 0 = -\tilde{b}_1 i_q - \tilde{b}_2\omega_e i_d - \tilde{b}_3\omega_e - \tilde{b}_2(\Delta i_{qdes}/\Delta t)$ *and* $0 = -\tilde{b}_1 i_d + \tilde{b}_2\omega_e i_q$. $\tilde{\theta} = \begin{bmatrix}\tilde{a}_1 & \tilde{a}_2 & \tilde{a}_3 & \tilde{b}_1 & \tilde{b}_2 & \tilde{b}_3\end{bmatrix}$ *and* $F_x^T = \begin{bmatrix}\omega & -1 & -\dot{\omega}_d & (-i_q - i_d) & (\omega_e i_q - \omega_e i_d - \Delta i_{qdes}/\Delta t) & -\omega_e\end{bmatrix}^T$ *can be described in matrix form. It follows that* $\tilde{\theta}F_x^T = 0$ *on M. Thus, the largest invariant set M in E is:*

$$M = \{(e, e_d, e_q, \tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{b}_1, \tilde{b}_2, \tilde{b}_3) \in R^9 | (e, e_d, e_q) = (0,0,0), \tilde{\theta}F_x^T = 0\}.$$

Hence, the adaptive controller (14), (18) and (22) with the update laws (27a–f) ensure that all the signals in the closed loop system are bounded and the equilibrium manifold $(\omega, i_q, i_d) = (\omega_d, i_{qdes}, i_{d\_d})$ *is globally asymptotically stable [19], or equivalently:*

$$\lim_{t\to\infty}\|[e(t), e_q(t), e_d(t)]\| = (0,0,0). \tag{29}$$

Proof. We set out to prove all the signals in the closed loop system are bounded and then tracking error signals are uniformly continuous. Consequently, we can demonstrate tracking error signals converge to zero as time goes to infinity by employing Barbalat's Lemma.

(**R1**) From the form of (28), $\dot{V}$ is negative or zero. This means that V is decreasing or constant; in other words, nonincreasing. V is also bounded from below by zero. Therefore, V has a limit as time goes to infinity: $0 \leqslant V(\infty) \leqslant V(0) < \infty$ for $\forall t \geqslant 0$ and thus $V(\infty) \in L_\infty$. This means $e, e_d, e_q, \tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{b}_1, \tilde{b}_2, \tilde{b}_3 \in L_\infty$ from (24). a_1, a_2, a_3, b_1, b_2, b_3 are constant from (6); hence, $\hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{b}_1, \hat{b}_2, \hat{b}_3 \in L_\infty$. Then, it is easy to show from (14) and (12) that i_{qdes}, i_q, i_{d_d}, $i_d \in L_\infty$. Using the assumption $\omega_d \in L_\infty$, it can be stated that $\omega \in L_\infty$ from (8). Owing to the fact that load torque disturbance is bounded by assumption, $\dot{\omega} \in L_\infty$ from the electromechanical dynamics of (7). $\dot{\omega}_d \in L_\infty$ by assumption, and consequently $\dot{e} \in L_\infty$.

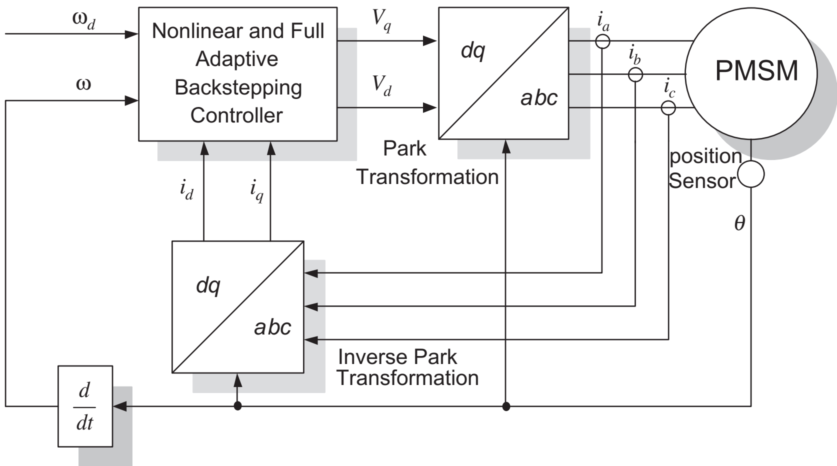

Fig. 1. Mathematical model of the overall control system.

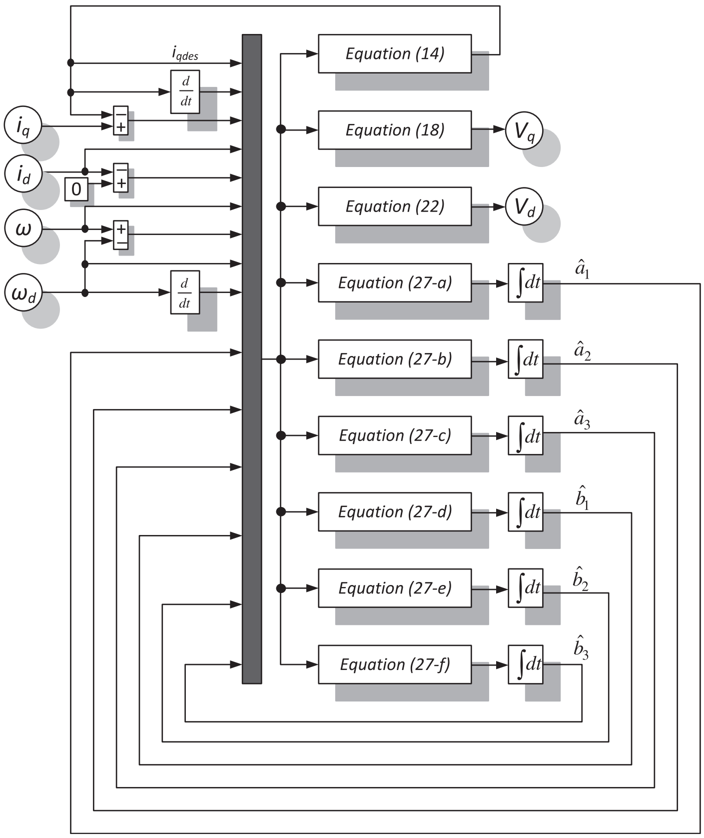

Fig. 2. Mathematical model of the nonlinear and full adaptive backstepping controller.

By taking time derivative of i_{qdes} from (14) analytically, we obtain: $\dot{i}_{qdes} = \left(\frac{1}{p}\right)(\hat{a}_1\dot{\omega} + \dot{\hat{a}}_1\omega + \dot{\hat{a}}_2 + \hat{a}_3\ddot{\omega}_d + \dot{\hat{a}}_3\dot{\omega}_d) - k_1\dot{e}$. If parameter update laws from (27) are substituted in this equation, we can show the analytic derivative of i_{qdes}"$\dot{i}_{qdes} \in L_\infty$" is bounded. Since the analytic derivative of any function is identical to the numerical derivative of that function, it is clear the numerical derivative of i_{qdes} "$(\Delta i_{qdes}/\Delta t) \in L_\infty$" is bounded. With the all of these, we can demonstrate that V_q and $V_d \in L_\infty$ from (18) and (22) respectively. Considering the electromechanical dynamics of (7), it is visible that $\dot{i}_q, \dot{i}_d \in L_\infty$. These imply that all the state variables with time derivatives are bounded. Then it is easy to understand that $\dot{e}_q$, $\dot{e}_d, \in L_\infty$. With this proof, it is proven that all the signals in the closed loop system are bounded.

(**R2**) Since $\dot{e}$, $\dot{e}_q$, $\dot{e}_d \in L_\infty$, e, e_q, e_d are uniformly continuous.

Now we continue to prove that the tracking error signals $e, e_d, e_q \in L_2$. With this aim, we integrate both sides of (28) from 0 to $+\infty$ and obtain:

$$-\int_0^\infty \dot{V}(t)dt = \int_0^\infty \left(k_1 e^2(t) + k_2 e_q^2(t) + k_3 e_d^2(t)\right)dt. \tag{30}$$

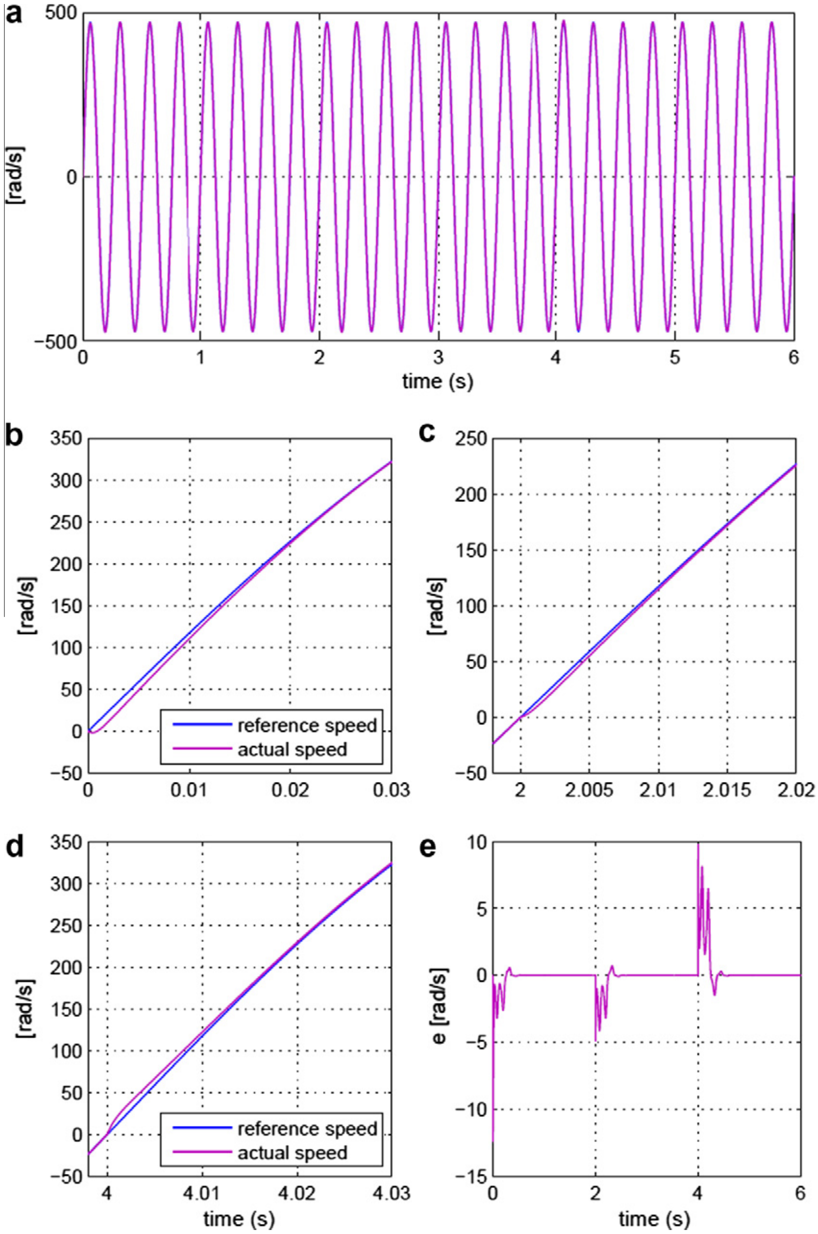

Fig. 3. Speed tracking response of the proposed controller to sinusoidal reference under load torque disturbance variation: (a) actual and reference velocities (b) zoom around 0 s (c) zoom around 2 s (d) zoom around 4 s (e) speed tracking error.

It follows that:

$$\int_0^\infty \left(k_1 e^2(t) + k_2 e_q^2(t) + k_3 e_d^2(t)\right) dt = V(\mathbf{0}) - V(\infty), \tag{31}$$

which becomes:

$$\sqrt{\int_0^\infty (k_1 e^2(t)) dt} \leqslant \sqrt{\int_0^\infty \left(k_1 e^2(t) + k_2 e_q^2(t) + k_3 e_d^2(t)\right) dt} \leqslant \sqrt{V(\mathbf{0})} < \infty, \tag{32}$$

which can be repeated for e_d and e_q as well. This proves that the integral of (30) exists and is finite since e, e_d, $e_q \in L_2$ [36]. Using Barbalat's Lemma, it can be demonstrated that if a signal $x(t)$ is uniformly continuous and $\lim_{t\to\infty} \int_0^t x(t)dt$ exists and is finite, then $x(t) \to 0$ as $t \to \infty$. Another way to say this is that if $\dot{x}(t) \in L_\infty$ and $x(t) \in L_\infty \cap L_2$, then $\lim_{t\to\infty} |x(t)| = 0$. Therefore, it is easy to verify the argument given in (29) is valid as a result of $\dot{e}$, $\dot{e}_d$, $\dot{e}_q \in L_\infty$ and e, e_d, $e_q \in L_\infty \cap L_2$ [35]. This implies that the error dynamics (e, e_d, e_q) converge to zero as time goes to infinity, while the equilibrium (0,0,0) in these dynamics remains

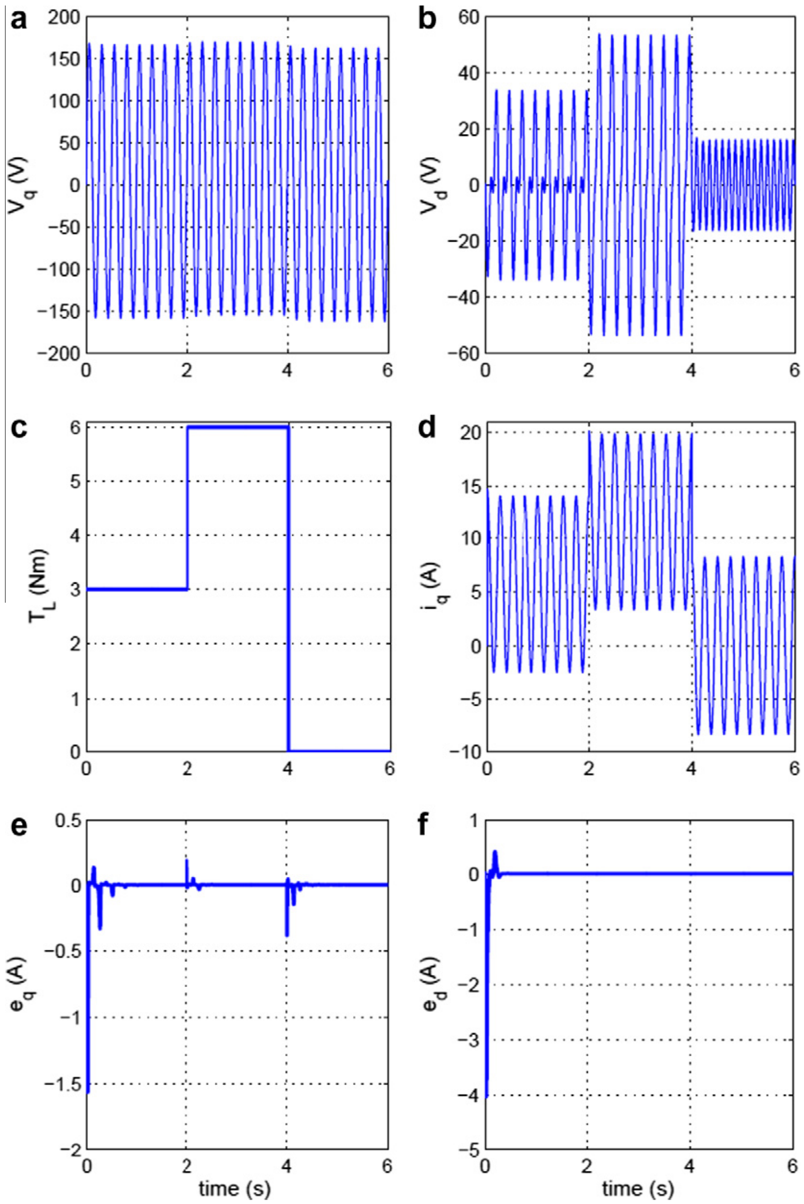

Fig. 4. Trajectories of the control inputs and tracking errors for sinusoidal reference under the load torque disturbance variation (a) trajectory of V_q (b) trajectory of V_d (c) variation of T_L (d) trajectory of i_{qdes} (e) trajectory of $e_q q$ (f) trajectory of e_d.

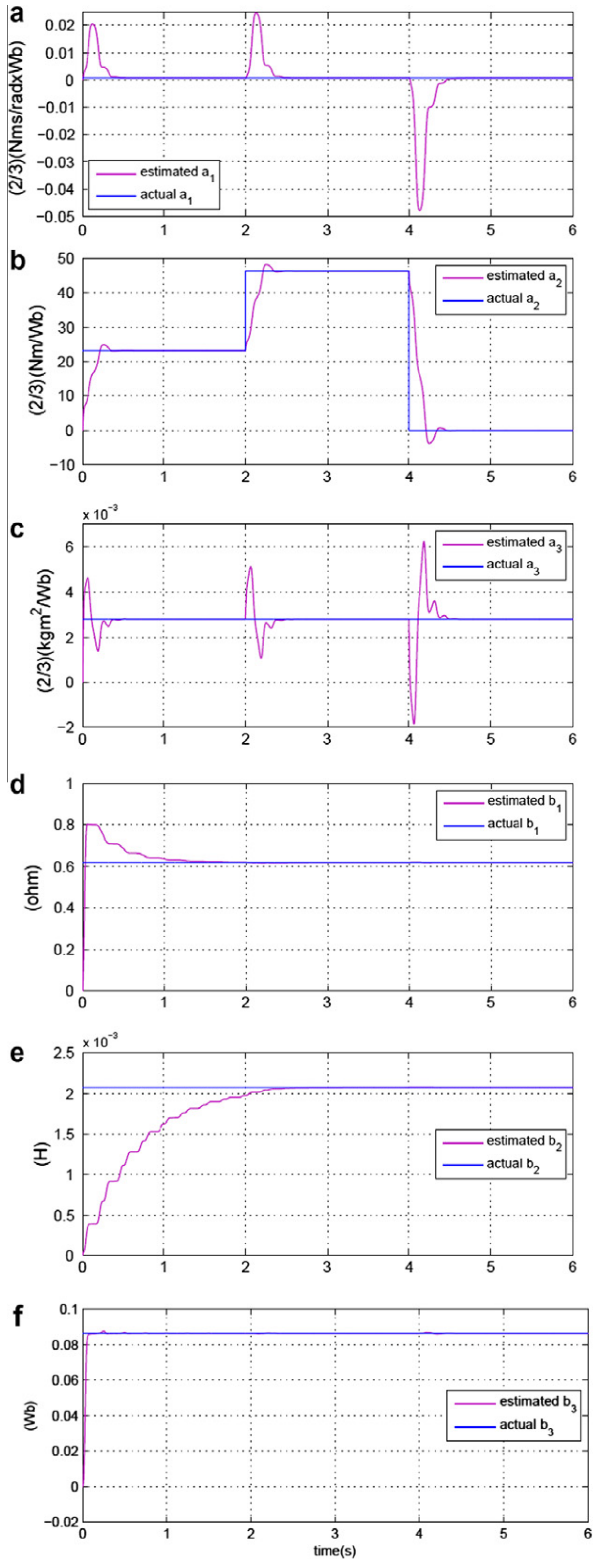

Fig. 5. The parameter and load torque disturbance estimations with their own actual values (a) estimation of a_1, (b) estimation of a_2 , (c) estimation of a_3 , (d) estimation of b_1 , (e) estimation of b_2 , (f) estimation of b_3.

globally asymptotically stable. This is the proof of the fact that global asymptotic speed tracking objective is achieved under the parameter uncertainties and unknown bounded load torque disturbance, whatever the initial conditions. Moreover, choices of the feedback and adaptation gains are arbitrary except for being positive and finite. That is, there is no inequality

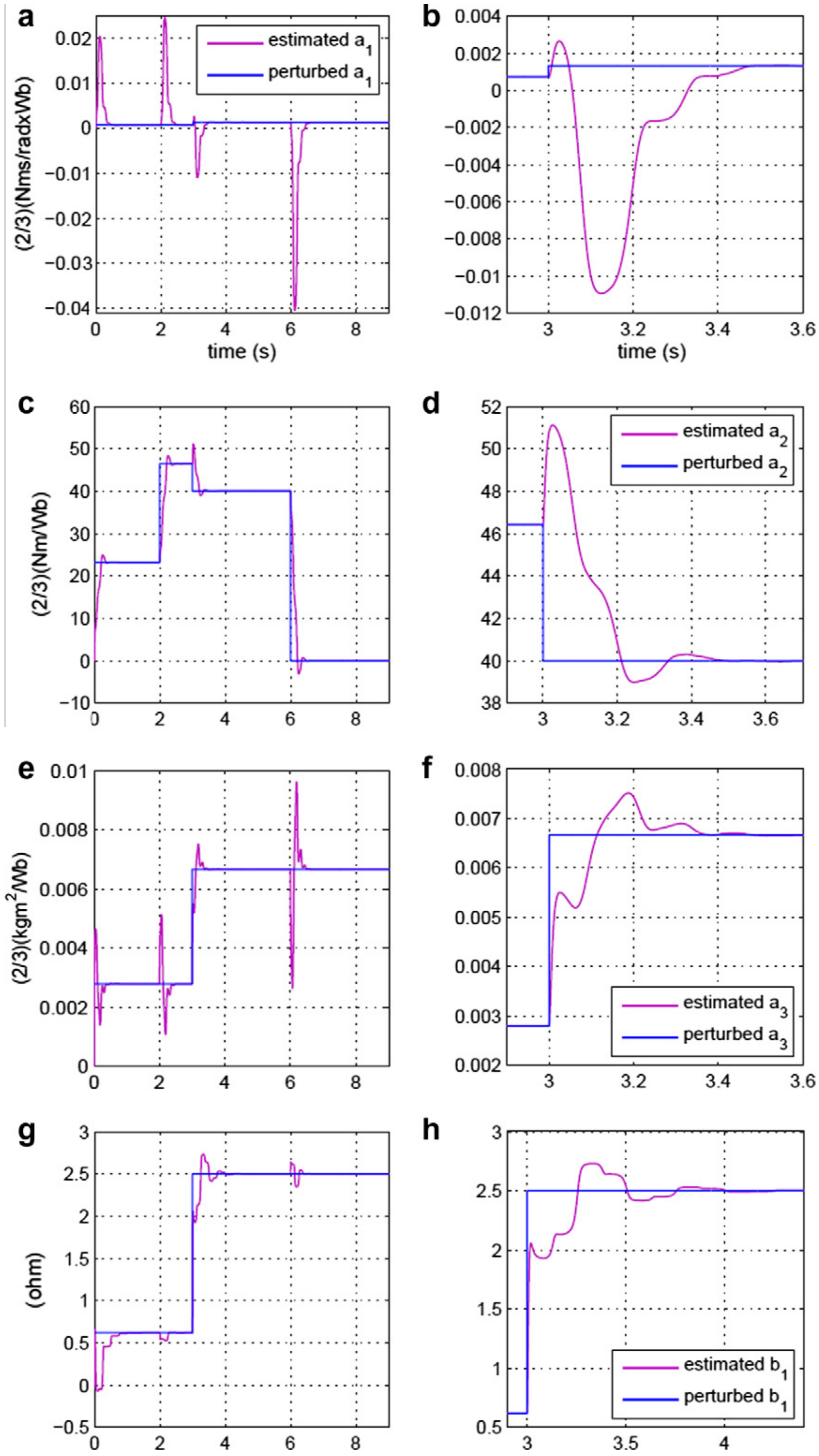

Fig. 6. Parameter estimating performance of the proposed controller for perturbed uncertain PMSM (a) perturbed and estimated a_1 (b) zoom around 3 s (c) perturbed and estimated a_2 (d) zoom around 3 s (e) perturbed and estimated a_3 (f) zoom around 3 s (g) perturbed and estimated b_1 (h) zoom around 3 s (i) perturbed and estimated b_2 (j) zoom around 3 s (k) perturbed and estimated b_3 (l) zoom around 3 s.

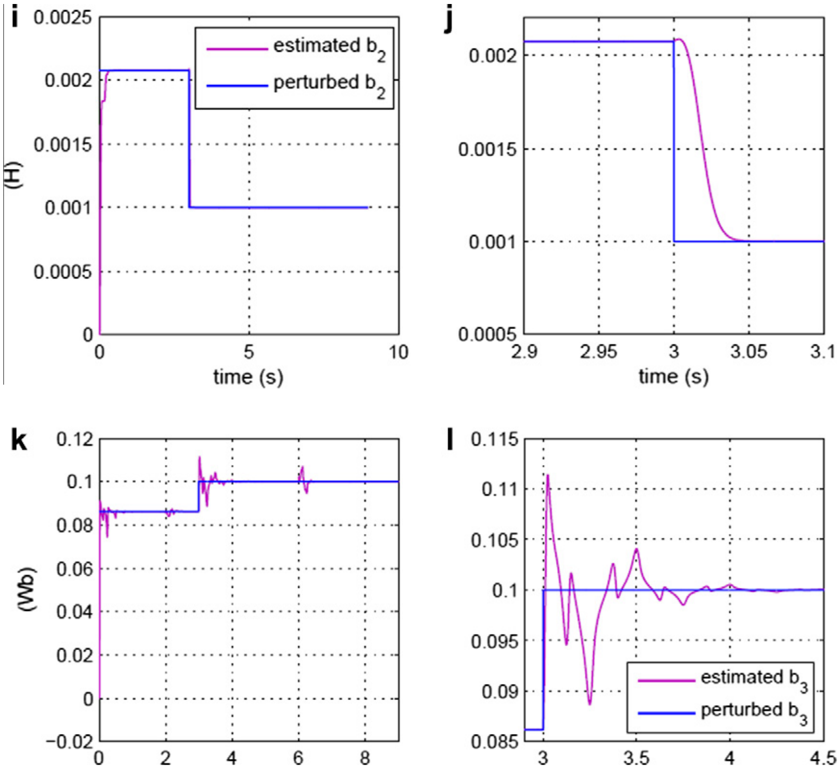

Fig. 6 (*continued*)

that the feedback gains need to satisfy; for this reason, asymptotic stability of the closed loop system is completely independent of the choices of these gains. It should be noted that any positive finite feedback gains stabilize the system.

3.3. Mathematical modeling of the overall control system and backstepping controller

The closed mathematical model of the overall control system and the detailed mathematical modeling of the nonlinear and full adaptive backstepping controller are given in the Figs. 1 and 2 respectively.

As seen in the Fig. 1, the state variables, i_q and i_d, are obtained by means of the fact that the park transformation is applied to three phase currents. And then, the controller algorithm given in the Fig. 2 can be carried out after the third state variable, ω, are obtained by taking time derivative from the knowledge of rotor position acquired by a position sensor. In this way, the control inputs V_q and V_d and virtual control input i_{qdes} are calculated through the Eqs. (14), (18) and (22) respectively.

As viewed in Fig. 2, the parameter estimations are computed by using Eq. (27a–f). The computed estimations then are feedback to the backstepping controller. After the inverse park transformation is applied to the control inputs, the three phase voltages stabilizing the error dynamics are applied to PMSM. This procedure is called the control loop that has to cyclically be performed with fixed time intervals. It is worth mentioning that frequency of the control loop should be selected as high as possible and, in turn, the accuracy of all the computations accordingly increases.

4. Simulation results and discussion

The data belonging to a PMSM with sinusoidal flux distribution used in the simulation are given as follows: Rated power; 2800 W, rated speed; 4500 rpm, DC voltage; 300 V, per phase inductance; 0.002075 H, per phase resistance; 0.62 Ω, rotor moment of inertia; 0.0003617 kg m^2, friction factor; 0.00009444 Nms, magnetic flux; 0.08627 Wb and number of pole pairs; 4. The value of the feedback and adaptation gains chosen are $k_1 = 1$, $k_2 = 25$, $k_3 = 5$ and $\theta_1 = 0.5$, $\theta_2 = 100$, $\theta_3 = 0.1$, $\theta_4 = 5$, $\theta_5 = 0.2$ and $\theta_6 = 1$ respectively. Besides, all the initial values of the parameter estimates are taken equal to zero in the simulations in order to observe the exact adaptation performance.

Case 1: For $\omega_d = 471\ sin(2\pi ft)$, $f = 4$ Hz and 6 s simulation time, disturbance rejection capacity and speed tracking performance of the controller are examined in detail under the load torque disturbance variation for the whole operating range including the region around zero. The load torque suddenly applied to PMSM is 3 Nm between 0–2 s, 6 Nm between 2–4 s and 0 Nm between 4–6 s. The dynamic speed tracking responses of the control systems corresponding to the loading conditions are given in Fig. 3a–f. The reference speed trajectory and actual speed are in Fig. 3a and their zooms about 0, 2 and 4 s as well as speed tracking error are given in Fig. 3b–e respectively. The Fig. 3 demonstrates that asymptotic speed tracking objective is achieved with high accuracy under load torque disturbance variation and parameter uncertainty.

In Fig. 4, the control inputs V_q and V_d, virtual control input i_{qdes} and d-q axis current tracking errors are provided. Since the reference speed trajectory is in sinusoidal form, the control inputs, V_q and V_d, in turn is in the same form. In the Fig. 3d, it is observed that the virtual control input has also sinusoidal form despite constant step load torque disturbance. The reason for this is that the lumped disturbance, the total disturbance effect on the PMSM resulting from the viscous friction and external load torque disturbance, is in sinusoidal form. As can be seen from the Fig. 4, precise control performance is achieved during both transient and steady state times; all the tracking errors remain at zero stably, against load torque disturbance variation. Fig. 5 plots the parameter estimations with the actual values of them. The initial values for all the estimates are taken equal to zero. As can be observed the Fig. 5, all the parameter estimates converge to their true values.

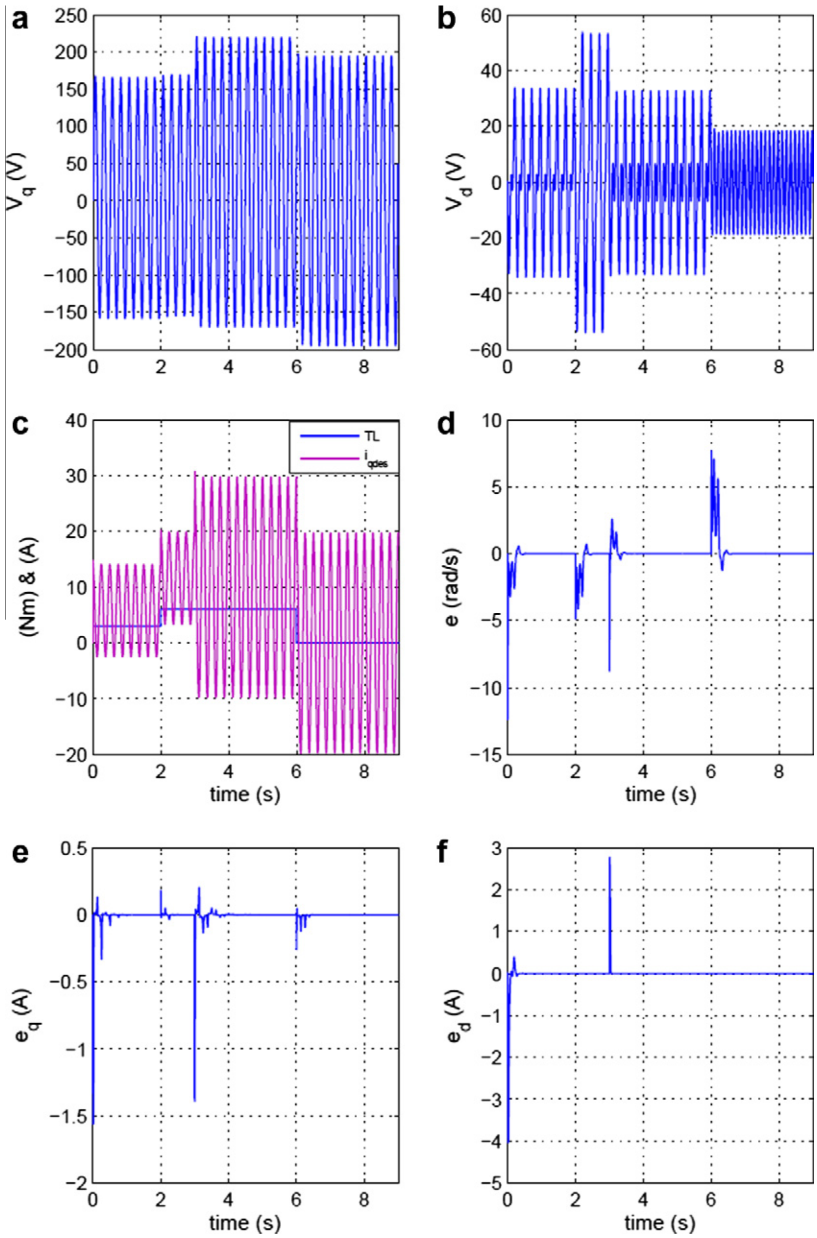

Fig. 7. Trajectories of the control inputs and tracking errors for sinusoidal reference under the perturbed PMSM model (a) trajectory of V_q (b) trajectory of V_d (c) variation of T_L and i_{qdes} (d) trajectory of e (e) trajectory of e_q (f) trajectory ofe_d.

Case 2: For $\omega_d = 471\ sin(2\pi ft)$, $f = 4$ Hz and 9 s simulation time, the robustness of the controller against the parameter perturbations introduced to the mathematical model of PMSM during simulation are examined under the load torque disturbance variation. The parameter perturbations introduced to the PMSM in the control are set to the following values; increasing from 0.62 Ω to 2.5 Ω in the stator resistance per phase, 0.08617 Wb to 0.1 Wb in the permanent magnet flux, 0.00009444 Nms to 0.0002 Nms in the viscous friction factor, 0.0003617 kg m^2 to 0.001 kg m^2 in the rotor moment of inertia and decreasing from 0.002075H to 0.001H in the stator inductance per phase at 3 s. These strong perturbations enable us to examine how the proposed controller responds to different operating conditions. Therefore, different operating conditions of real world applications are reflected in the simulations properly. All the parameter estimations with zoom of them in convergence times are given in Fig. 6, which reflect that all the parameter estimations converge to their true values as in Case 1. This is because of the fact that the reference speed trajectory, ω_d, is chosen to be sufficiently rich signal satisfying the persistency of excitation condition [33,34]. Fig. 7 shows the variation of the control inputs V_q and V_d, virtual control input i_{qdes} and d-q axis current tracking errors. As can be seen in the Fig. 7, owing to the fact that all the control inputs rapidly converge to the values stabilizing the error dynamics, all the tracking errors remain at zero stably. As can be observed from the simulation results, it is obvious that the proposed speed tracking controller guarantees strong robustness with regard to all the parameter uncertainties/perturbations and unknown bounded load torque disturbance in both PMSM and load dynamics respectively.

5. Concluding remarks

In this study, a new nonlinear and full adaptive backstepping speed tracking controller design is developed for an uncertain PMSM. Except for the number of pole pairs, all the other parameters and load torque disturbance are considered uncertain and adaptively estimated. The proposed controller ensures global asymptotic tracking of a time varying reference speed and robustness against all the uncertainties in both PMSM and load dynamics respectively. The simulations results validate feasibility and applicability of the proposed controller. With the novelty which is concluded in the item ii. and iv below, the achievements of this study can be summarized in the following order:

(i) In order to deal with mismatched and nonlinearly parameterized uncertainties, several artificial intelligence based, in other words approximator based, adaptive backstepping control methods have been reported. The promising ones of them are adaptive backstepping neural network control [32] and adaptive backstepping fuzzy logic control [28]. The proposed methods do not use the assumption of linear-in-the-parameters and regression matrices; however, it is worthy to note that the increasing complexity resulting from combining the concepts of artificial intelligence and adaptive backstepping becomes an important shortcoming. In this study, the proposed controller does not require computation of the regression matrices. Therefore, has relatively less complex formulation than the methods above, which mean that the (D1) is overcome.

(ii) Except for the number of pole pairs, all the parameters and load torque disturbance in PMSM drive system are considered uncertain and so adaptively estimated, which eliminate the (D2). As to the number of pole pairs, it is nameplate information and does not vary with different operating conditions; correspondingly, it is not essential for it to be estimated adaptively. Since the reference speed trajectory is chosen as sufficiently rich in frequencies, it is persistently exciting [33,34]. Thus the estimated parameters converge to their true values, and this case is verified by the simulation results as well. The proof of persistence of excitation is given in detail in [37]. Besides, not only uncertain but also perturbed uncertain PMSM mathematical models are utilized in the simulations. For both situations, it is shown that the parameter estimation errors along with the tracking errors converge to zero.

(iii) The linearization theorems are not used in the design of the controller, the (D3) is eliminated.

(iv) The final time derivative of the Lyapunov function does not include any positive definite term; which implies that there is no limitation on choices of the feedback gains. Consequently, it is clearly deduced from that the asymptotic stability of the overall control system does not depend on that the feedback gains satisfy any inequality or condition. That means the (D4) is eliminated. But, it should be noted the feedback gains are responsible for the tracking performance while the adaptation gains specify adaptation performance.

(v) The number of the estimations in the new design is equal to the number of uncertain parameters in the PMSM drive system. Besides, none of the estimations appears as a denominator of any control input. Hence, it is obvious that the proposed controller overcomes the (D5), overparameterization and singularity problems.

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